ULTRANET AND PROPERTIES OF NET

(Dr.S.SANGEETHA,M.MAHALAKSHMI,P.PRIYA,P.ELAVARASI)
(sangeethasankar2016@gmail.elavarasi30com,@gmailcom)
Department of Mathematics
Dhanalakshmi Srinivasan College of
Arts and Science for Women (Autonomous)
Perambalur

ABSTRACT

In this paper we deals with ultranet and properties of net. Every net has a subnet which is altranet. That is a net in a topological space that is residually in every set or it's complement in the space If an ultranet has x as a cluster point, then it converges to x. Then the convergences of net play fundamental rate in the topological space

INTRODUCTION

A net x in a set x is an ultranet iff each subset E of X. That if an ultranet is frequently in E , then it is exsidually in E. In particular ultranet is a topological space must convergence each of its cluster points .Every subset of an ultranet is ultranet.

KEYWORDS

Ultranet, subnet Hausdraff space , Filters

DEFINITION

A net $\{f_a : a \in A\}$ in a set X is said to be an ultra net (or universal net) if and only if for each subset S of X, the net is eventually in S or eventually in S^I.

THEOREM

Let $\{f_a : a \in A\}$ be an ultra net in X and g be a mapping of X into a set Y. Then $\{g(f_a) : a \in A\}$ is an ultra net in Y.

PROOF

Let S be any subset of Y. Then it is easy to see that $g^{-1}[S] = X - g^{-1}[Y\text{-}S]$.

Since $\{f_a\}$ is an ultra net in X and $g^{-1}[S]$ is a subset of X with $g^{-1}[Y\text{-}S]$ as its complement, it follows from the previous definition that $\{f_a\}$ is eventually in either f^{-1} [S] or f^{-1} [Y-S], from which it follows that {g (f_a)} is eventually in either S orY-S.

Hence {g (f_a)} is an ultra net in Y.

THEOREM

Let {s (n): n ∈ D} be an ultra net in X and let g: X → Y. Then {(g o s) (n):n∈D}is an ultra net in Y.

PROOF

Let S ⊂ Y. Then g^{-1}(S)=g^{-1}[Y-(Y-S)]

=g^{-1}(Y)-g^{-1}(Y-S)

=X-g^{-1} (Y-S).

Thus, [g^{-1}(S) $]^0$=g^{-1}(Y-S).

Since {s (n):n∈D} is an ultra net in X, it is eventually in either g^{-1}(S) or g^{-1}(Y-S)

So, ∃ n ∈ D such that

$m \in D, m \geq n \Rightarrow s(m) \in g^{-1}(s)$

$\Rightarrow s_m \in g^{-1}(Y - S)$

$\Rightarrow g[s(m) \in S \; or \; g[s(m)] \in (Y - S)$

$\Rightarrow (g \; o \; s)\,(m) \in S \; or \; (g \; o \; s)(m) \in (Y - S)$

$\Rightarrow \{(g \; o \; s)\,(n): n \in D\}$

is eventually in S or eventually in Y-S. Hence $\{(g \; o \; s)(n): n \in D\}$is an ultra net inY.

THEOREM

Let (X, □) be a topological space. A point $x_o$in X is a cluster point of a net (f, X, A, ≥) iff there exists a subnet (g, X, B, ≥*) which converges to x_0.

PROOF

NECESSARY POINT

Assume that f has a subnet g which converges to x_o.

TO PROVE THAT

x_o is a cluster point of f.

Let N be □-neighborhood of x_o and let a_o be any element of A. Since g is a subnet of f, there exists a mapping ϕ: B → A suchthat

□g = f o ϕ and

□for each α in A, there exist an element b in B such that □(x) ≥ a for every $x \geq^* b$ in B.

Hence by second condition, corresponding to $a_o \in A$, there exists an element $b_o \in B$ suchthat

$$\varphi(x) \geq a_o \text{ for every } x \geq^* b_o.$$

Since g converges to x_o, there exists an element $p \geq^* b_o$ in B such that g (p) ∈ N.

Now let $\varphi(p) = q$.

$$Then\ q \in A\ and\ q \geq a_0.$$

$Also\ f(q) = f(\varphi(p))$

$$= (f\ o\ \varphi)(p)$$

$$= g(p) \in N.$$

Thus we have shown that for each element a_o in A, there exists an element $q \geq a_o$ in A such that f (q) ∈ N.

Hence f is frequently in N. It follows that x_o is a cluster point of f.

SUFFICIENT PART

Let x_o be a cluster point of a net (f, X, A, ≥) and let N (x_o) be the collection of all □-neighborhoods of x_o. If L, M is any two members of N (x_o), then L ∩ M is also a member of N(x_o).

Also since $x_o$is a cluster point of f, f is frequently in each member of N(x_o).

Hence by the previous theorem there exists a subnet g of f which is eventually in each member of N (x_o). This implies that g converges to *x*.

COROLLARY

Let (A, ≥) be a directed set and B be a cofinal subset of A so that B is also directed by ≥ Let (f, X, A, ≥) be a net. Then the restriction map of f to B is a subnet off.

PROOF

Here the identity map

I: B → A; I (*x*) = *x*

is an isotone mapping such that [B] = B is co final in A. Hence by
the above theorem f o I is a subnet of f.
Since I is a mapping of B into A and J is mapping of A into X it follows that f o I is a mapping of B into X such that (f o I) (*x*) = f (I *x*)) =f(*x*) for all *x* ∈ B. Hence f o I is the restriction map of f to B. It follows that the restriction map of f to B is a subnet off.

NOTE

The type of subnet in the above corollary is called a co finite subnet.

PROPERTIES

Virtually all concepts of topology are often rephrased within the language of nets and limits. This may be useful to guide the intuition since the notion of limit of nets is very similar to that of limit of a sequence.
A function f : X → Y between topological spaces is continuous at the point x iff
for every net (x_α) with

$$\lim x_\alpha = f(x)$$

we have $\lim f(x_\alpha) = f(x)$

This theorem is generally not true if we replace “net” by “sequence”.

RELATION TO HAUSDORFF SPACE:

In general, a net during a space X can have quite one limit, but if X may be a Hausdorff space, the limit of a net, if it exists, is unique.

Conversely, if X isn't Hausdorff, then there exists a net on X with two distinct limits. Thus the uniqueness of the limitis equivalent to the Hausdorff condition on the space, and indeed this may be taken as the definition. This result depends on the directedness condition; a set indexed by a general preorder or partial order may have distinct limit points even in a Hausdorffspace.

If U may be a subset of X, then x is within the closure of U if and as long as there exists a net (xα) with limit x and such xαis in U for allα.A subset A of X is closed if and only if, whenever (x_α) is a net with elements in A and limit x, then x is inA.

The set of cluster points of a net is equal to the set oflimits of convergentsubnets.

A net has a limit iff all of its subnets have limits. In that case, every limit of the net is also a limit of everysubnet.

A space X is compact if and only if every net (x_α) in X has a subnet with a limit in X.This can be seen as ageneralization

of the Bolzano-Weierstrass theorem and Heine-Borel theorem.

A net in the product space has a limit if and only ifeach projection has alimit. Symbolically, if (xα) may be a net within the iXi , then it converges to x if andπproduct X = as long as for every i. Armed with this observation and the above characterization ofcompactness

in terms on nets, one can give a proof of Tychonoff's theorem.
If $f : X \to Y$ and $(x\alpha)$ is an ultra net on X, then $(f(x\alpha))$ is an ultra net onY.

RELATION TO FILTERS

A filter is another idea in topology that allowed for a general definition foe convergence in general topological spaces. The two ideas are equivalent within the sense that they provide an equivalent concept of convergence. More specifically,for every filter base an associated net are often constructed, and convergence of the filter base implies convergence of the associated net- and therefore the other way around (for every net there is a filter base, and convergence of the filter base). For instance, any net in induces a filter base of tails where the filter in generated by this filter base is called the net's eventually filter. This correspondence allows for any theorem that can be proven with one concept to beproven with the other. For instance, continuity of a function from one topological space to the other can be characterized either by the convergence of the corresponding net in the co domain, or by the same statement with filter bases.

Robert G. Bartle argues that despite their equivalence, it's useful to possess both concepts. In any case, he shows how the 2 are often utilized in combination to prove various theorems generally topology.

APPLICATIONS

Topology is employed in many branches of mathematics like differentiable equations, dynamical systems, knot theory, and Riemann surfaces in complex analysis. It is also utilized in string theory in physics, and for describing the space- time structure of universe. They are also used in biology, computer science, physics, robotics, fiber art, games and puzzles. Some of the applications are:

BIOLOGY

Knot theory, a branch of topology, is employed in biology to review the consequences of certain enzymes on DNA. These enzymes cut, twist and reconnect the DNA, causing knotting with observable effects like slower electrophoresis.

PHYSICS

Topology is relevant to physics in areas such as condensed matter physics, quantum field theory and physical cosmology. In cosmology, topology are often wont to describe the general shape of the universe. This area of research is commonly known as space time topology.

ROBOTICS

The possible positions of a robot are often described by a manifold called configuration space. In the area of motion planning, one finds paths between two points in configuration space. These paths represent a motion of the robot’s joints and other parts into the specified pose.

GAMES AND PUZZLES

Tanglement puzzles are supported topological aspects of the puzzle’s shapes and components.

FIBER ART

In order to make endless join of pieces during a modular construction, it's necessary to make an unbroken path in an order which surrounds each bit and traverses each edge only once. This process is an application of the Eulerian path.

CONCLUSION

In this desertion we discussed about related topics about convergence of net basic definition and result of convergence used in theorems.

The concept of convergence of net is explained lucidly with the notion of sequence and physical applications. Further this physical application through this chapter help one in the advanced reading of the subject which is becoming more and moreabstract.

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