TRAVELLING SALESMAN PROBLEM

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ABSTRACT

Travelling salesman problem can be modeled as graph where as the cities are the graph vertices, path is graph edges and path distance is edge distance. Our goal is to seek out the shortest tour that visits each city during a given graph exactly ones then return to the starting city

KEYWORDS

Spanning tree, weighted graph, dots, vertex, edges, and Hamilton cycle.

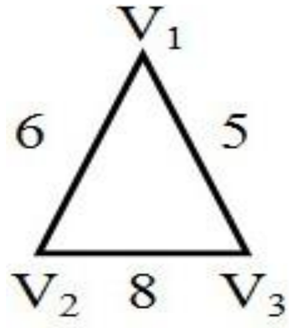
INTRODUCTION

The traveling sales man problem appeared in rudimentary from in practical germen book of 1831. Princeton University in the 1930's the traveling salesman problem was mathematically formulated in 1800's by the Irish mathematician W.R Hamilton and by British mathematician Thomas Kirk man. Hassles Whitney at Princeton University introduced the name travelling salesman problem soon afterward one method of doing this was to create a minimum spanning tree of the graph.

DEFINITION:

A Graph G is claimed to be a weighted graph. If there's a true number related to each fringe of G.

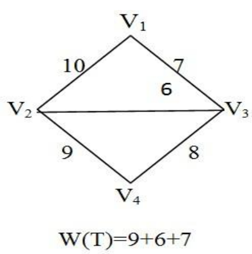
Example

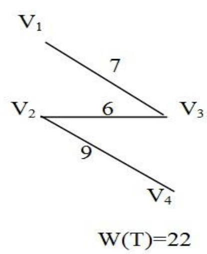


DEFINITION

If a graph is weighted graph than the load of a Spanning tree T of it's defined because the sum of this weights of all the branches in T and is defined by N (T).

Example

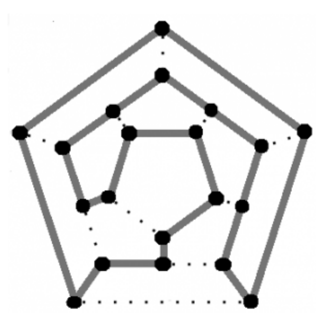




DEFINITION:

A Connected graph is Hamiltonian if it contains a cycle that includes every vertex such a cycle is a Hamilton cycle.

Example

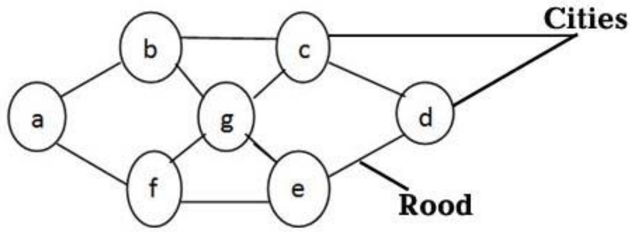


DEFINITION:

A traveler wishes to find a tour that visits each city exactly once and returns to the starting point.

Example:

We find the travelling



The traveler wishes to find to tour that starts at city a goes to each city exactly once and ends back at a; Two examples of such a tour area b c d e g f a and a f e d c g b

The traveler visits each city just once but may omit several roads.

Let us regard the road map as a connected graph whose vertices correspond to the cities and whose edges correspond to the roads.

The traveler's problem is now to find a cycle that includes every vertex of the graph.

Problem:

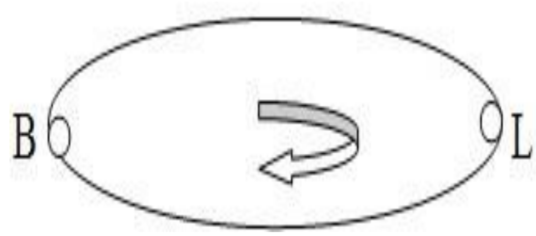
	Berlin	London	Moscow	Pari
Berlin	-	7	1 1	7
London	7	-	1 8	3
Moscow	1 1	1 8	-	1
Paris	7	3	1 8	-
Rome	1 0	1 2	2 0	9
Seville	1 5	1 1	2 7	8

We find an boundary for the answer to the traveling salesman problem for the six European cities.

First Vertex

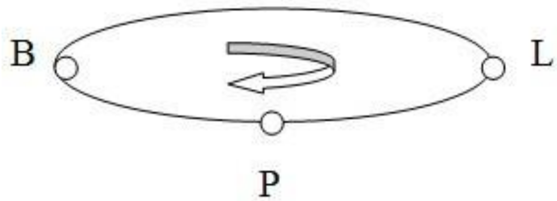
We insert Seville ahead of Paris within the cycle.

Second Vertex



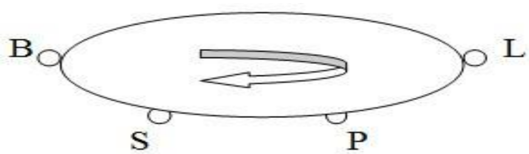
The city nearest to Berlin is London or Paris (distance 7). Let us choose London. We draw two vertices and two edges joining them, and give the 'cycle' a clockwise orientation.

Third Vertex



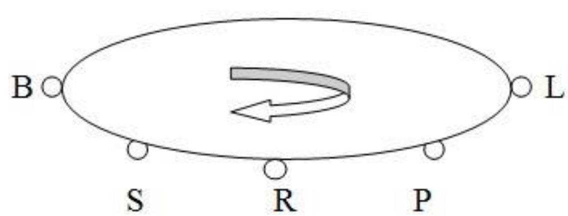
The city nearest to Berlin and London is Paris (distance 3 from London).
We insert Paris in front of London in the cycle.

Fourth Vertex



The city nearest to Berlin, London and Paris is Seville (distance 8 from Paris).
We insert Seville in front of Paris in the cycle.

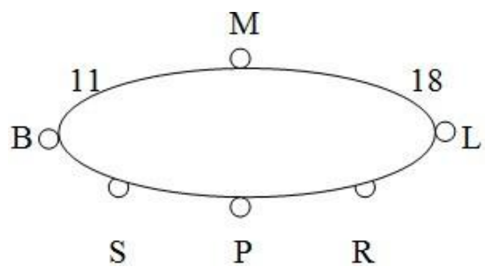
Fifth vertex



The city nearest to Berlin London, Paris and Seville is Rome (distance 9 from Paris.

We insert Rome ahead of Paris within the cycle.

Sixth vertex

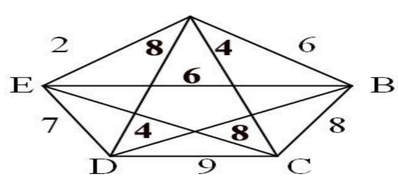


The final city is Moscow. It is closest to Berlin (distance 11). We insert Moscow ahead of Berlin within the cycle.
Berlin – Moscow – London – Paris – Rome – Seville – Berlin An boundary for the answer is therefore

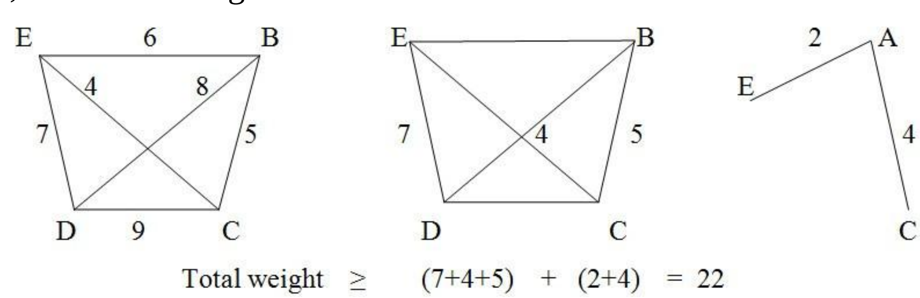
$$11 + 18 + 3 + 9 + 13 + 15 = 69$$

Problem:

We find a boundary for the answer to the traveling salesman problem for the subsequent weighted graph.



We start by removing any vertex. Let us choose the vertex x A. Then the remaining weighted graph has the four vertices B, C, D, and E.
The minimum spanning tree joining these vertices is that the tree whose edges are ED, CE, and BC, with total weight



The minimum spanning tree joining these vertices is the tree whose edges are ED, CE, and BC, with total weight

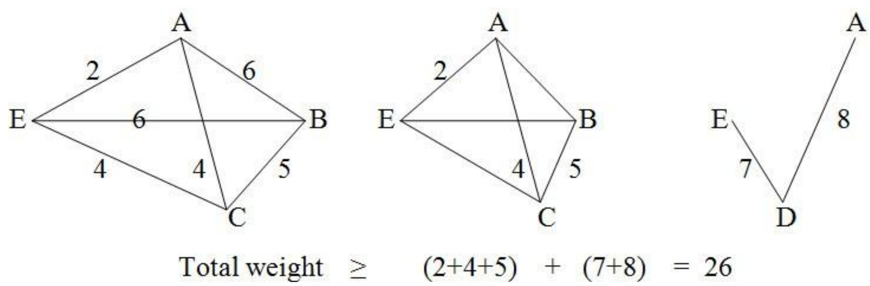
$$7 + 4 + 5 = 16.$$

The two edges of smallest weight incident with A are AE and AC, with weights 2 and 4.
A lower bound for the solution is therefore

$$16 + 2 + 4 = 22$$

A little experimentation shows that this boundary isn't very good: the particular solution to the present problem is that the cycle ACBDEA with total weight 26, so our boundary is not the correct solution.
A better lower bound (that is a greater one, giving more information) can be obtained by removing the vertex D instead of A. In this case, the remaining weighted graph has the four vertices A, B, C, E, and there are two minimum spanning trees joining these vertices, each with total weight

$$2 + 4 + 5 = 11.$$



The two edges of smallest weight incident with D are DE and DA, or DE and DB, with weights 7 and eight . A better lower bound for the solution is therefore 11 + 7 + 8 = 26, which is the correct solution.

Theorem:

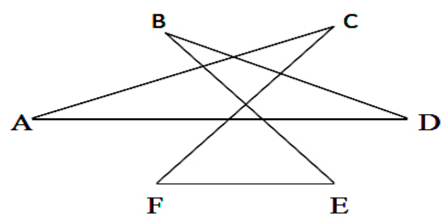
If a graph G satisfies the ore property, then G contains a Hamiltonian Cycle.

Proof:

Assume, on the contrary, that for a few integer $n \geq 3$, there exists a graph H of order n such for every pair of nonadjacent vertices u and v, deg u + deg v $\geq$ n and yet H isn't Hamiltonian. Add as many edges to the graph of H as possible between pair of nonadjacent vertices so as that the resulting graph G remains not Hamiltonian.
(If we add the other edge, G are going to be Hamiltonian. In other words, G may be a Maximal non – Hamiltonian graph). By the idea that we made about H, we

all know that for each pair of non adjacent vertices u and v in G, degGu + degGv $\geq$ n. Also note that G isn't an entire graph.

C' = (x = v1, v2, ….Vi-1, y = vn, vn-1, Vn-2, …., vi, x) may be a Hamiltonian cycle of G, which is impossible. Hence, for every vertex of G adjacent to x, there's a vertex of V (G)\ {y} not adjacent to to y. However then, degGy $\leq$ (n-1) – degGX degGX + degGY $\leq$ (n-1) which may be a contradiction. Note that ore's Theorem isn't a necessary condition for a Hamiltonian Cycle, but a sufficient condition.
Figure doesn't satisfy Ore's Property since the sum of the degrees of each pair of nonadjacent vertices is 4, which is a smaller amount than 6. However, the graph still contains a Hamiltonian Cycle, namely
(A → C → F → E → B → D →A).



This problem is said to Hamiltonian circuits suppose a salesman has got to visit variety of cities [each city features a road to each other city] during his trip. Given the distances between the cities. In what order should the salesperson travel so on visit every city precisely once and return to his home city with the minimum mileage travelled?

Solution

(i) Represent the cities by vertices, and therefore the roads between them by edges. Then we get a graph. during this graph far every edge e there corresponds a true number we (the distance in miles say) such a graph is named a weighted graph. Here we are called because the Weight of the sting .
(ii) If each of the cities features a road to each other city, we've an entire weighted graph. This graph has numerous Hamiltonian circuits and that we need to select the Hamiltonian circuits that have the littlest sum of distances (or weight).
(iii) The number of various Hamiltonian circuits (may not be edge – disjoint) in complete graph of n vertices is adequate to (n-1)!/2.

Theorem:

Let G be an easy graph with n vertices, and let U and V be the non – adjacent vertices in G such d(u) + d(v) $\geq$ n

Let G + UV denote the super graph of G obtained by joining U and V by jittery .
Then G is Hamiltonian ↔ G + UV Hamiltonian

Proof:

It is clear that G is Hamiltonian → its super graph G + UV of G is Hamiltonian

Converse:

Suppose that G + UV is Hamiltonian. during a Contrary way suppose that G isn't Hamiltonian. As within the proof of above theorem. We get that d (u) + d (v) < n a contradiction to the hypothesis that d(u) + d(v) ≥ n. This shows that G must be Hamiltonian.

Dirac 1952 If may be a simple graph with n vertices where n ≥ 3 and d(v) ≥ n/2 for each Vertex V of G then G is Hamiltonian

CONCLUSION

The travelling salesman problem is one among the challenging problems within the real world and also most well studied combinatorial optimization problem. the thought of travel sales man problem has much application in from different fields like operational research, algorithms design and including AI attract by it

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