CLUSTER POINT OF NET AND ISOLATED MAPPING

(Dr.S.SANGEETHA,P.ELAVARASI,R.RAMYA,M.MAHALAKSHMI)
(sangeethasankar2016@gmail.com,elavarasi30@gmailcom)
Department of Mathematics
Dhanalakshmi Srinivasan College of
Arts and Science for Women (Autonomous)
Perambalur

ABSTRACT

In mathematics, more specifically generally topology and related branches, a net or Moore-Smith sequence may be a generalization of the notion of sequence. In essence, a sequence may be a function with domain the natural numbers and within the context of topology, the co domain of this function is typically any mathematical space . However, within the context of topology, sequences don't fully encode all information a few function between topological spaces.

INTRODUTION

Nets are one of the tools used in topology to generalize certain concepts that may only be general enough in the context of metric spaces. A related notion that of the filter was developed in 1937 by Henri Cartan.

The term “net” was coined by John L. Kelley. The purpose of the concept of a net, first introduced by E. H. Moore and Herman L. Smith in 1922, is to generalize the motion of a sequence so as to confirm the equivalence of the conditions. In particular, this enables theorems almost like that asserting the equivalence of conditions, to carry within the context of topological spaces that don't necessarily have a countable or linearly ordered neighbourhoods basis around a point.

In this chapter we shall discuss about the topic of convergence of net in a topological space.

Keywords

- Isotone mapping
- Cluster point
- Neighborhood

Preliminaries

DEFINITION:

TOPOLOGICAL SPACE

If $\mathfrak{I}$ is a topology on X, then (X, $\mathfrak{I}$) is called a topological space.

DEFINITION:

Let X be a non – empty set and let (D, ≥) be a directed set. Then, a mapping s : D → X is called a net in X.

DEFINITION:

DEFINITION:

ISOTONE MAPPING

Let $(D, \geq)$ and $(E, \geq^*)$ be two directed sets. Then, a mapping $\phi : E \rightarrow D$ is claimed to be isotone iff

$$m \geq^* n$$

$$\Rightarrow \phi(m) \geq \phi(n)$$

$Where\; m, n \in E.$

DEFINITION:

CLUSTER POINT

Let (X, $\mathfrak{I}$) be a mathematical space and let f be a net in X. Then a point $x_o \in$ X is called a cluster point of net f iff f is frequently in every $\mathfrak{I}$-neighborhood of $x_{o.}$

EXAMPLE:

Let N be directed by ≥ in the usual sense. Consider the mapping f : N → R defined in such a way that f(N) = Q. Then f may be sequence and hence a net in R.

Now we will deduce the isotone mapping and cluster point of a net.

CLUSTER POINT OF A NET

THEOREM

Let (ϕ, X, D, ≥) be a net and let ψ be an isotone mapping of (E, ≥*) into (D, ≥) such that ϕ (E) is co final in D. Then ϕ o ψ is a subnet ofϕ.

PROOF:

Let f = ϕ o ψ so that f: E → X in order that ϕ : D → X and ψ : E → D.

Therefore ψ is also a net in X.

Further we are given that ψ is an isotone mapping of E into D and hence by definition

a ≥ *b ⇒ ψ (a) ≥ ψ (b), (a,b) ∈ E.

But we are as long as ψ (E) may be co final set in D in order that by definition for every a in D there exists a component b in E such that $\psi(x)$ ≥ a for every x ≥ *b.

Therefore $\phi \, o \, \psi$ is a subnet of ϕ.

THEOREM

Every convergent net in a Hausdorff space has a unique cluster point which is the unique point to which the net converges.

PROOF:

Let (X, τ) be a Hausdorff space. Then every convergent net in X will converge to a unique point.

Let $(\{S_n : \in D\}, \geq)$ be a net converging to a point $x \in X$. Then by convergence, ∃ n ∈ D such that

For each m ∈ D, m ≥ n ⇒ s_m ∈ N for every neighborhood N of *x*.

In particular, for each n ∈ D ∃ $n \geq n$ in D such that $s_n \in N$ for every neighborhood N of *x*.

This shows that *x* is a cluster point of the net $(\{S_n : \in D\}, \geq)$.

Now, if we choose any point y ∈ X, distinct from *x*, then X being a Hausdorff space ∃ neighborhoods N and M of *x* and y respectively such that N $\cap M = \phi$.

Since x is a cluster point of the net ({s_n : n ∈ D}, ≥), it follows that for each n ∈ D ∃ m ≥ n in D such that $s_m \in N$ and therefore, $s_m \notin$ M, since N ∩ M = ϕ.

Thus, y is not a cluster point of $(\{S_n : \in D\}, \geq)$

Hence $(\{S_n : \in D\}, \geq)$ can not have more than one cluster point.

THEOREM

Let (X, τ) be a mathematical space and $let\ (f, X, A, \geq)$ be a net in X. For each a in A, let $M_a = \{f\ (x) : x \geq a\ in\ A\}$. Then a point p of X is a cluster point of f iff $p \in \overline{M_a}$ for all a ∈ A.

PROOF:

NECESSARY PART:

Let $p \in \overline{M_a}$ ∀ a ∈ A and suppose, if possible, p is not a cluster point of f.

Then there exists a neighborhood N of p and an element a in A such that $f\ (x) \notin N$ for every x ≥ a in A.

This implies that $N \cap Ma = \emptyset$.

It follows that $p \in \overline{M_a}$ for this a.

This is a contradiction.

Hence p is a cluster point of f.

SUFFICIENT PART:

Let p be a cluster point of f and let N be an arbitrary neighborhood of p.

Then f is usually in N, that is, for every a ∈ A, there exists $x \geq a$ during a such $f\ (x) \in N$.

$Hence\ M_a \cap N \neq \emptyset\ for\ each\ a \in A.$

Thus each neighborhood of p intersects M_a for all a ∈ A.

It follows that $p \in \overline{M_a}$ for every a ∈ A.

CONCLUSION

In this desertion all of about cluster point of net in mathematical space .Then the convergence of net play fundamental role within the mathematical space . during this applications to varied fields like Biology, computing , Physics, Robotics, Games and puzzles, Fiber art.

The concept of convergence of net is explained lucidly with the notion of sequence and physical applications. Further this physical application through this chapter help one within the advanced reading of the topic which is becoming more and more abstract.

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