FREE GROUPS AND FREE PRODUCTS OF GROUPS

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ABSTRACT:

One can use van Kampen's theorem to calculate fundamental groups for topological spaces that can be decomposed into simpler spaces. Thus we can see that there is a commutative diagram including A∩B into A and B and then another inclusion from A&B into s^2 and that there is a corresponding diagram of homomorphism b/w the fundamental groups of each subspace. It is clear from this that the fundamental group of s^2 is trivial.

KEYWORDS:

free product, the external free product, free group, free group on the element betti number.

INTRODUCTION:

Herbert karl Johannes Seifert & van Kampen introduced the problem of describing the fundamental group of a space X in terms of the fundamental groups of the constituents x_i of an open covering.In mathematics, the Seifert-van Kampen theorem of Algebraic topology, sometimes it is called as van Kampen's theorem. It expresses the structure of the fundamental group of a topological space X in terms of the fundamental groups of two open, path connected subspaces u and v that covers X.

DEFINITION:

Let G be a group, let $\{G_\alpha\}_{\alpha \in J}$ be a family of subgroup of G that generate G. Suppose that $G_\alpha \cap G_\beta$ consists of the identity element alone whenever $\alpha \neq \beta$. We say that G is the **free product** of the groups G_α if for each $x \in G$, there is only one reduced word in the groups G_α that represents x. In this case, wewrite

$$G = \prod^{*}_{\alpha \in J} G_\alpha$$

or in the finite case, $G = G_1 * G_2 * \cdots * G_n$.

DEFINITION:

Let $\{G_\alpha\}_{\alpha \in J}$ be an indexed family of groups. Suppose that G is a group, and that $i_\alpha \colon G_\alpha \to G$ is a family of monomorphisms, such that G is the free product of the groups (G_α). Then we say that G is the **external free product** of the groups G_α, relative to themonomorphisms i_α.

DEFINITION:

If S is a subset of G, one can consider the intersection N of all normal subgroups of G that contains S. It is easy to see that N is itself a normal subgroup of G; it is called the **least normal subgroup** of G that contains S

DEFINITION:

Let $\{a_\alpha\}$ be a family of elements of a groups G. Suppose each a_α generates an infinite cyclic subgroup G_α of G. If G is the free product of the groups $\{G_\alpha\}$, then G is said to be a **free group**, and the family $\{a_\alpha\}$ is called a **system of free generators** for G.

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DEFINITION:

Let $\{a_\alpha\}_{\alpha \in J}$ be an arbitrary indexed family. Let G_α denote the set of all symbols of the form a_α^n for $n \in Z$. We make G_α into a group by defining

$$a_\alpha^n . a_\alpha^m = a_\alpha^{n+m}$$

Then a_α^0 is the identity element of G_α, and a_α^{-n} is the inverse of a_α^n. We denote a_α^1 simply by a_α. The external free product of the groups $\{G_\alpha\}$ is called the **free group on the elements** a_α.

DEFINITION:

The rank of H is uniquely determined by G, since it equals the rank of the quotient of G by its torsion subgroup. This number is often called the **betti number** of G.

Lemma:3.3

Let {□□} be a family of groups; let G be a group; let □□: □□→ □be a family of homomorphisms. If each □□is a monomorphism and G is the free product of the groups (□□), then G satisfies the following condition:

Given a group H and a family of homomorphisms (∗) h □: □□→ □, there exists a homomorphisms h : □ →□ such that h ∘ □□= h □for each □.

Furthermore, h is unique.

Theorem:3.4 (Uniqueness of free products).

Let {□□}α□J be a family of groups. Suppose G and□|

are groups and □$_□$: □$_□$→ □and □$'$: □$_□$→ □$'$are families of

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monomorphisms, such that the families {□$_□$(□$_□$)}

and

{□$'$(□)}generateGand□$'$respectively.IfbothGand□$'$

□ □

have the extension property stated in the in the preceding lemma, then there is a unique isomorphism □: G → □$'$such that □ ∘ □$_□$= □$'$for all □.

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CONCLUSION:

In this dissertation we have discussed some basic definition. Also we have discussed the direct sum of abelian groups, Free Products of Groups and Free Groups. We also deals with the major theorem “The Seifert-van Kampen theorem” of the dissertation.

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