APPLICATIONS OF VECTOR ALGEBRA IN SPHERE AND FORCE

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Abstract:

This "Introduction to vector- methods and their various applications to physics and mathematics" is an exposition of the late Willard Gibbs' vector analysis. The author in his preface warns us that "no attempt at mathematical rigor is made" which perhaps explains the opening sentence of chapter "A vector is any quantity having direction as well as magnitude." What of finite rotations? Are they not to be considered quantities having direction and magnitude? In an appendix the author compares notations, not always quite accurately. He believes Willard Gibbs' notation to be the simplest and most symmetrical of any of the existing kinds. Burali-Forti and Marcolongo, who believe they have devised the perfect notation, object to Willard Gibbs' "dot" in the scalar product, using a "cross" instead. As regards the question of symmetry, the truth is that the vector product is not symmetrical, for in Gibbs's notation $a \times b = a$. As a matter of fact each vector analyst can always find sufficiently self-pleasing arguments in favour of his pet notation.

Introduction

In our day to day life, we come across many quires such as. What is your height? How a football player should hit the ball to give a pass to another player to his team? Observe that a positive answer to the first query may be 1.6 meters, a quantity that involves only one value (magnitude) which is a rod number. Such quantities are scalars.

Vector algebra is introduced in (1805-1865) by W.R. Hamilton.

However, an answer to the second query is a quantity (called force) which involves muscular strength (magnitude) and direction n (in which another player is positioned). Such quantities are called vectors. We frequently come across with both types of quantities, namely, scalar quantities volume, temperature, work, money, voltage, density resistance etc... And vector quantities like displacement, velocity, acceleration, force, weight, momentum electric field about vectors.

We will study some of the basic concepts aboutvectors various operations on vectors, and their algebraic and geometric properties. These two types of properties, when considered to gather give full realizations to theconcepts of vectors, and lead to their vital applicability in various areas as mentioned above.

In this chapter we shall discuss about the topic of applications of vector algebra in vector equations and straight line.

Key words:

- Vector algebra
- Spatial
- Force
- Magnitude
- Position vector
- Equilibrium

Definition:

Vector algebra:

An algebra for which the elements involved may represents vectors and the assumptions and ruler are based on the behaviour of vectors.

Definition:

Spatial:

A vector $\vec{x}$ is said to be spatial if it lies in a space.

Definition:

Position vector:

We take in space any arbitrary point O, to be called as the origin of reference.

Then the position vector of any point P, with respect to the origin O is the vector$\overrightarrow{OP}$.

We usually denoted the position vector of the points A, B, C by $\vec{a}, \vec{b}, \vec{c}$ respectively.

Definition:

If the magnitude of a scalar quantity is constant, then it is known as constant scalar.

Definition:

If the magnitude and direction of a vector quantity is constant, then it is known as constant vector.

Definition:

SPHERE:

A sphere is the locus of a variable point in space whose distance from a fixed point in the space is always a constant. The fixed point is known as the centre of the sphere and the constant length of any point on the sphere from the centre is known as its radius.

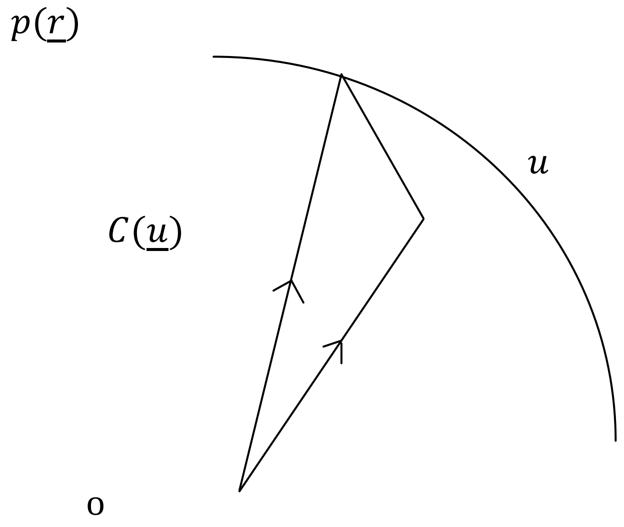
Now we will deduce the equations of sphere under different condition.

4.1 Sphere with Given Radius and Centre

we consider a sphere whose centre is at C with position vector $\vec{u}$ relative to a fixed point O chosen as origin. Let P be any point with position vector $\vec{r}$ on the surface of the sphere whose radius is of ρ units. Then we have $\overrightarrow{CP} = \vec{r} - \vec{u}$. Now $\left|\overrightarrow{CP}\right|^2 = \rho^2$. Hence we have

$$|\vec{r} - \vec{u}|^2 = \rho^2 \Longrightarrow |\vec{r}|^2 + |\vec{u}|^2 - 2\vec{r}.\vec{u} = \rho^2, \quad \ldots\ldots (4.1)$$

Which is the required equation of the sphere.



Corollary 1:

If the centre of the sphere is at the origin then the equation of the sphere reduces to the following:

$$|\vec{r}|^2 = \rho^2, \text{ i.e., } |\vec{r}| = \rho \qquad \ldots\ldots (4.2)$$

Corollary 2:

If the sphere passes through the origin O, then we have $\left|\overrightarrow{OC}\right| = |\vec{u}| = \rho$ and hence the equation of the sphere reduces to the following:

$$|\vec{r}|^2 - 2\vec{r}.\vec{u} = 0. \qquad \ldots\ldots (4.3)$$

FORCE

We know that a force is an action which when applied on a body will either change or tend to change the state of rest or of uniform motion of the body in a straight line. A force has magnitude as well as definite direction. Thus a force is a vector quantity. Which can be completely represented by a directed line segment. It is an example of a localized vector quantity and its physical effect on a body is altered if it is shifted to a parallel position even though its magnitude and direction remain unaltered. Thus a single free vector cannot completely represent the effect of the force on the body but can do so only in respect of magnitude and direction. Hence at this stage we restrict ourselves to forces whose lines of action meet at a point. i.e; the forces which act on a single particle. These forces are called concurrent forces.

Equilibrium of Coplanar Concurrent Forces

THEOREM: A

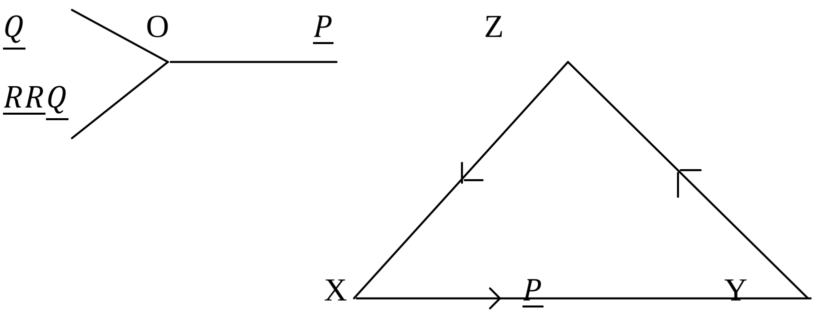
If three coplanar concurrent forces acting at a point on a body can be represented in magnitude and direction but not in position by the three sides of a triangle taken in order, then the forces are in equilibrium and conversely,

PROOF:

We consider that the forces $\vec{P}, \vec{Q}, \vec{R}$ acting at the point O be represented in magnitude and direction by the sides XY, YZ, ZX of a triangle XYZ, taken in order. Then from the adjoining figure it is found that

$$\vec{P} + \vec{Q} + \vec{R} = \overrightarrow{XY} + \overrightarrow{YZ} + \overrightarrow{ZX} = \overrightarrow{XY} + \left(\overrightarrow{YZ} + \overrightarrow{ZX}\right) = \overrightarrow{XY} + \overrightarrow{YX} = \vec{0},$$

Thus the forces are in equilibrium.



Now we prove the converse part as follows:

The resultant of $\overrightarrow{XY}$ and $\overrightarrow{YZ}$ is $\overrightarrow{XZ}$. Since the forces $\vec{P}, \vec{Q}, \vec{R}$ are in equilibrium, so the force $\vec{R}$ is equal and opposite to that of the resultant of the forces $\vec{P}$ and $\vec{Q}$. Hence $\vec{R}$ must be represented in magnitude and direction by $\overrightarrow{ZX}$. Therefore, the sides of the triangle considered in order represents the forces $\vec{P}, \vec{Q}, \vec{R}$. Hence the theorem follows.

NOTE:

The above law is known as the triangle law of forces force! triangle law.

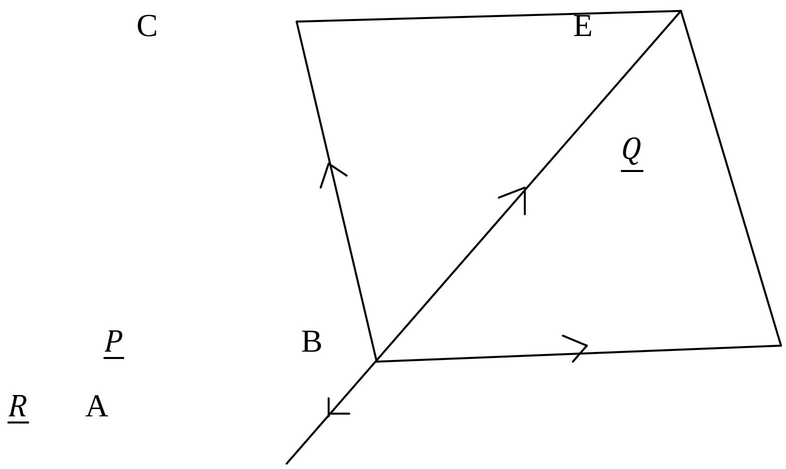
THEOREM: B

If three coplanar forces acting at a point on a body be in equilibrium, then each force is proportional to the since of the angle between the other two forces.

[This is known as Lami's theorem]

PROOF:

We consider that the force $\vec{P}, \vec{Q}, \vec{R}$ acting at A be represented in magnitude and direction by AB, AC, AD respectively and also the forces are in equilibrium. Let P, Q, R be the magnitude of the forces $\vec{P}, \vec{Q}, \vec{R}$ respectively. We now draw the parallelogram ABED



so that the diagonal $\overrightarrow{AE}$represents the resultant of $\vec{P}$ and $\vec{Q}$. Since the forces are in equilibrium so the force $\vec{R}$ and $\overrightarrow{AE}$ are equal and opposite. Now we have

$sin\angle\, AEB = sin\angle\, CAE = sin\ sin(\pi - \angle\, CAD) = sin\angle\, CAD,$

$sin\angle\, BAE = sin\ sin(\pi - \angle\, BAD) = sin\angle\, BAC,$ and,

$sin\angle\, ABE = sin\ sin(\pi - \angle\, BAC) = sin\angle\, BAC,$

Thus from the triangle $\triangle\, ABE$ we have

$$\frac{AB}{sin\,\angle AEB} = \frac{BE}{sin\,\angle BAE} = \frac{AE}{sin\,\angle ABE},$$

Or, $$\frac{AB}{sin\,\angle CAD} = \frac{AC}{sin\,\angle BAD} = \frac{AD}{sin\,\angle BAC},$$

Or, $$\frac{P}{sin\,\angle CAD} = \frac{Q}{sin\,\angle BAD} = \frac{R}{sin\,\angle BAC}$$

Since $P{:}\,Q{:}\,R = AB{:}\,AD{:}\,AC$

Conclusion:

If this paper we discussed about related topics about vector algebra basic definition and vector equation used in straight line theorems. Then also this desseration all of about applications of vector algebra in sphere and force.

Then the vector algebra play fundamental role in the straight line, plane and sphere. In this applications to various fields such as geometry, mechanics, physics, and engineering, scientific research.

The concept of vector algebra is explained lucidly with the geometric notations and physical motivations. Further the geometrical and physical applications through this chapters help one in the advanced reading of the subject which is becoming more and more abstract.

Conclusion

1. R. Gupta “Golden math series vector algebra” by Laxmi publications (p) ltd, New delhi-110055.
2. A.P. Wills: vector analysis with an introduction to tensor analysis, Dover publications, Inc., New York,
3. A.R. Vasishtha “vector algebra” 17th edition 1966-1997, Krishna prakashan media (p) Ltd, Meerut-250001 (U.P).
4. C.J. Eliezer: concise vector analysis, The Macmillan Company, New York, 1961.
5. Absos Ali sheikh: “vector analysis with applications” narosa publishing house pvt. Ltd, new delhi-110 002.