A STUDY ON SEPARATION AXIOMS

K.CHENDRAN[1],R.PREMKUMAR[2],M.MAHALAKSHMI[3],A.KIRUTHIKA[4]

Department of Mathematics
Dhanalakshmi Srinivasan College of
Arts and Science for Women (Autonomous)
Perambalur

ABSTRACT:

Topology is a beautiful science and forms a bridge between geometry and algebra.Topology means (Topo-place,logy-study) i.e.,study of place.One of the most wiedly used mathematical concepts in the study of place,optimization of any machinery parts such as mobiles,cycles,computers,cars,buses,trains and airplanes,etc.In this paper,the concept of a separation axioms considered.The separation axioms are only in the sense that when defining the notation of topological space.The separation axims are denoted with the letter "T" after the German Trennungsaxiom,which means "Separation Axioms".In this paper proposed T_0,T_1,T_2,T_3 and T_4 axioms with examples.

KEYWORDS:

Open set,Closed set,Normal Space,Regular Space,Topological Space.

1.INTRODUCTION:

Separation axioms are statements about the richness of a topology.we wil see that the higher separation axioms have other interesting characteristic as well.We denote these conditions by T_0,T_1,T_2,T_3 and T_4 is increasing the order of richness.

2.PRELIMINARIES:

DEFINITION 2.1:

Let X be a non-empty set and $\Im$ be the collection of subsets of X. Then $\Im$ is "topology" for X, if the following properties are satisfied.

I. $X \in \Im$ and $\varphi \in \Im$.

II. It is closed under the operation of finite intersection.

III. It is closed under the operation of arbitrary union.

The members of $\Im$ are called open sets of the topology $\Im$ and the pair $(X,\Im)$ is called a "topological spaces".

DEFINITION 2.2:

Let X be a non-empty set and $\Im$ is a topology on X then every member of $\Im$ is called an "open sets".

DEFINITION 2.3:

Let $(X,\Im)$ be a topological space. Then Let $(X,\Im)$ is said to be "regular space" if given an element $x \in X$ and closed set $F \subset X$ such that

$$x \notin F$$

There exist disjoint open sets $G, H \subset X$ such that

$$x \in G, F \subset H.$$

DEFINITION 2.4:

Let $(X,\Im)$ be a topological space. Then Let $(X,\Im)$ is said to be "normal space" if for every pair of disjoint closed sets $F_1, F_2 \subset X$. This implies there exist $\Im$-open sets G and H such that

$$F_1 \subset G, F_2 \subset H$$

$$G \cap H = \varphi$$

3.T_0-SPACE:

DEFINITION 3.1:

Let $(X,\Im)$ be a topological space. Then $(X, \Im)$ is said to be T_0-space if and only if for distinct points x_1 and x_2 in X there exist a $\Im$-open set G such that

$x_1 \in G$ and $x_2 \notin G$

or

$x_2 \in G$ and $x_1 \notin G$.

EXAMPLE 3.2:

Let X = {a,b,c} and $\Im$ = {X,φ,{a},{b},{a,b}}. To show that (X, $\Im$) is a T_0-space.

SOLUTION:

Given that X = {a,b,c}

$\Im$ = {X,φ,{a},{b},{a,b}}

Using the definition of T_0-space,

For distinct element a and b there exist a $\Im$-open set {a} (or) {b} such that

$a \in \{a\}$ and $b \notin \{a\}$

or

$b \in \{b\}$ and $a \notin \{b\}$

Hence (X, $\Im$) is a T_0-space.

4.T_1-SPACE:

DEFINITION 4.1:

A topological space (X,$\Im$) is said to be T_1-space if each singleton is closed.

Or

Let (X,$\Im$) be a topological space. Then (X,$\Im$) is said to be T_1-space if for each distinct pair x,y then there exist two open sets G and H such that

$x \in G$ but $y \notin G$

and

$y \in H$ but $x \notin H$.

EXAMPLE 4.2:

Let X= {1,2,3} and $\mathfrak{I}$ = {X,φ,{1},{2},{1,2}}. To show that (X,$\mathfrak{I}$) is a T_1-space.

SOLUTION:

Given that X= {1,2,3}and

$\mathfrak{I}$ = {X,φ,{1},{2},{1,2}}

Here 1 and 2 are two distinct pairs of X then there exist two open set {1} and {2} such that

$1 \in \{1\}$, $2 \notin \{1\}$

and

$2 \in \{2\}$, $1 \notin \{2\}$

Hence (X,$\mathfrak{I}$) is a T_1-space.

5.T_2-SPACE:

DEFINITION 5.1:

Let (X,$\mathfrak{I}$) be a topological space.Then (X,$\mathfrak{I}$) is said to be a T_2-space if for each distinct pair of element x and y there exist neighbourhood N and M such that

$x \in N, y \in M$ and $N \cap M = \varphi$.

EXAMPLE 5.2:

Let X={1,2,3} and $\mathfrak{I}$={X,φ,{1,2},{3}}.Then show that (X,$\mathfrak{I}$) is not a Hausdorff space.

SOLUTION:

Given that X={1,2,3} and

$\mathfrak{I}$={X,φ,{1,2},{3}}.

For a,b distinct elements of X there are no disjoint neighbourhoods.

Hence,the given (X,$\mathfrak{I}$) is not a Hausdorff space.

6.T_3-SPACE:

DEFINITION 6.1:

A regular T_1-space is known as T_3-space.

RESULT 6.2:

Every T_3-space is T_2-space.

PROOF:

We know that a regular T_1-space is called a T_3-space.

Let (X,$\mathfrak{I}$) be a T_3-space.

Let x,y be any two distict points of X.

Using the definition of T_3-space,

This implies X is also a T_1-space and so {x} is a closed set.

Also $y\notin\{x\}$.

Since X is a regular space.

This implies there exist open sets G and H such that

$\{x\}\subset G, y\in H$ and

$G\cap H=\varphi$.

Also

$\Rightarrow$x,y belong respectively two disjoint open sets G and H.

$\Rightarrow x\in G, y\in H, x\notin H, y\notin G$

And $G\cap H=\varphi$

i.e.,given space is T_2-space.

7.T_4-SPACE:

DEFINITION 7.1:

A normal T_1-space is known as T_4-space.

RESULT 7.2:

Let $(X,\Im)$ be a topological space.If $(X,\Im)$ is a T_4-space then it is also T_2-space.

PROOF:

Let $(X,\Im)$ be a T_4-space i.e.,

1.X is T_1-space and

2.X is normal space.

To show that X is T_2-space.

Let $x,y\in X$ be arbitrary such that $x\neq y$.Because X is T_1-space.

$\Rightarrow\{x\}$ and $\{y\}$ are disjoint closed sets in X.

Also X is normal space.

$\Rightarrow$given a pair of disjoint closed sets $\{x\},\{y\}\subset X$.

There exist disjoint open sets G and $H\in\Im$ such that

$\{x\}\subset G,\{y\}\subset H$

$x\in G,y\in H$.

i.e.,space is T_2-space.

Hence,T_4-space is also T_2-space.

8.CONCLUSION:

We conclude that T_0,T_1,T_2 are axioms that tells how well the open set can be separate points from each other.T_3 describes the ability of the open sets to separate points from closed sets.T_4 is the ability of the open sets to separate disjoint closed sets.

REFERENCES:

1. A.K Malik and S.R Singh, topology, I.K International Publishing House Pvt.Ltd.(2012).

2. A.R Singal and S.P Arya, some new separation axioms in topological ordered spaces,Math Student 40A(1972).

3. H.A Priestley, separation theorems for semi-continuous function on normally ordered topological spaces,J.London Math.soc.3(1971).

4. S.D McCartan, separation axioms for topological ordered space, Mat proc.Cambridge.Philos.soc.64(1968).

5. P.A Arya and Kavita Gupta, new separation axioms in topological ordered spaces,Indian J.Pure Appl.Math.22(1991).