VECTOR EQUATION AND STRAIGHT LINE

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ABSTRACT

We will study some of the basic concepts about vectors various operations on vectors, their algebraic and geometric properties. These two types of properties, when considered to gather give full realizations to the concepts of vectors, and lead to their vital applicability in various areas.

KEYWORDS

Scalar, Vector, Collinear vector, Like vector, Unlike vector, **coplanar**

INTRODUCTION

Vector algebra is introduced in (1805-1865) by W.R. Hamilton.

However, an answer to the second query is a quantity (called force) which involves muscular strength (magnitude) and direction n (in which another player is positioned). Such quantities are called vectors. We frequently come across with both types of resistance etc... And vector quantities like displacement, velocity, acceleration, force, weight, and momentum electric field about vectors.

Vectors are main objects of study in multivariable calculus. They have the same role that numbers have in single-variable calculus. It is very important to gain a solid understanding of vectors before proceeding to multivariable calculus.

DEFINITION:1.1

A **vector** is a quantity having both magnitude and definite direction, in space is called **vector quantities**.

Example:

Force, velocity, acceleration, displacement, momentum, etc., are all vectors.

DEFINITION: 1.2

A **scalar** is a quantity having magnitude but no direction in space is called **scalar quantities**.

Example:

Mass, length, volume, density, time, temperature, etc., are all scalars.

DEFINITIONS: 1.3

(i) Vectors having the same sense of direction are called **Like** Vectors**.**

(ii) Vectors having the opposite sense of direction are called Unlike vectors**.**

(iii) **Collinear vectors (or) Parallel vector** are two or more vectors which are parallel to the same line irrespective of their magnitude and direction.

(iv) Vectors, whose directions are neither co-incident nor parallel, are called **Non-collinear vector.**

(v) Three or more vectors are said to be c**oplanar** when they lie in a same plane or they are parallel to the same plane.

(vi) When there is no restriction to choose the origin of a vector, then it is called a **free vector**.

(vii) When there is restriction to choose a certain specified point, then it is called a **Localized vector**.

(viii) If the magnitude of a scalar quantity is constant, then it is known as **Constant Scalar.**

(ix) If the magnitude and direction of a vector quantity is constant, then it is known as **Constant Vector**.

VECTOR EQUATION:

An equation involving known and unknown scalar and vector quantities is known as the **Vector Equation**.

For example, $\boldsymbol{\lambda\vec{x} = \mu\vec{a}}$, where λ, μ are constant scalars, $\vec{a}$ is a constant vector and $\vec{x}$ is an arbitrary vector, is a **vector equation**.

NOTE:

In the sequel we use the notations $\vec{a}, \vec{b}, \vec{c}$ etc. for constant vectors and $\vec{x}, \vec{y}, \vec{z}$ etc. for non-constant vectors, unless otherwise stated.

We now state and prove some theorems concerning vector equations.

THEOREM: 1.1

The vector equation $\boldsymbol{\lambda\vec{x} + \mu\vec{a} = v\vec{b}}$, where $\lambda \neq 0, \mu$ and v are constant scalars and $\vec{a}, \vec{b}$ are constant vectors, has a unique solution $\boldsymbol{\vec{x} = \frac{1}{\lambda}\left(v\vec{b} - \mu\vec{a}\right).}$

PROOF:

Firstly, we suppose that the given equation admits a solution. Therefore, we must have

$$\boldsymbol{(\lambda\vec{x} + \mu\vec{a}) - \mu\vec{a} = v\vec{b} - \mu\vec{a}}$$

Or, $\lambda\vec{x} + (\mu\vec{a} - \mu\vec{a}) = v\vec{b} - \mu\vec{a}$, by associative law

Or, $\lambda\vec{x} + \vec{0} = v\vec{b} - \mu\vec{a}$

Or, $\frac{1}{\lambda}\lambda\vec{x} = \frac{1}{\lambda}(v\vec{b} - \mu\vec{a})$

Or, $\vec{x} = \frac{1}{\lambda}(v\vec{b} - \mu\vec{a})$.

Secondly, substituting $\frac{1}{\lambda}(v\vec{b} - \mu\vec{a})$ in place of $\vec{x}$ in the given equation we obtain

$$\lambda\vec{x} + \mu\vec{a} = \lambda\left\{\frac{1}{\lambda}\left(v\vec{b} - \mu\vec{a}\right)\right\} + \mu\vec{a}$$

$$= \left(v\vec{b} - \mu\vec{a}\right) + \mu\vec{a}$$

$= \vec{b} + (-\mu\vec{a} + \mu\vec{a})$, by associative law

$$= v\vec{b} + \vec{0}$$

$$= v\vec{b}$$

Thus $\boldsymbol{\vec{x}} = \frac{\mathbf{1}}{\boldsymbol{\lambda}}(\boldsymbol{v\vec{b}} - \boldsymbol{\mu\vec{a}})$ is a solution of the given equation.

Next we prove the uniqueness of the above solution. To prove it, if possible, let x_1 and x_2 be two solutions of the given equation. Hence we have

$$\lambda\vec{x}_1 + \mu\vec{a} = v\vec{b} \;\; and \;\; \lambda\vec{x}_2 + \mu\vec{a} = v\vec{b} \; .$$

Comparing the last two equations we get

$$\lambda\vec{x}_1 + \mu\vec{a} = \lambda\vec{x}_2 + \mu\vec{a}$$

$$\lambda\vec{x}_1 + \mu\vec{a} - (\lambda\vec{x}_2 + \mu\vec{a}) = \lambda\vec{x}_2 + \mu\vec{a} - (\lambda\vec{x}_2 + \mu\vec{a})$$

$\lambda\vec{x}_1 - \lambda\vec{x}_2 + \mu\vec{a} - \mu\vec{a} = \vec{0}$, by associative, distributive and commutative law

$\lambda(\vec{x}_1 - \vec{x}_2) = \vec{0}$, by distributive law

$\vec{x}_1 - \vec{x}_2 = \vec{0}$, since $\lambda \neq 0$

$$\vec{\boldsymbol{x}}_{\mathbf{1}} = \vec{\boldsymbol{x}}_{\mathbf{2}} \ .$$

We now state and prove a theorem on vector equation involving known vectors but unknown scalars.

THEOREM: 1.2

The vector equation $\boldsymbol{\lambda}\vec{\boldsymbol{x}} + \boldsymbol{\mu}\vec{\boldsymbol{y}} = \vec{\boldsymbol{z}}$, where λ and μ are unknown constant scalars $\vec{x}, \vec{y}$ and $\vec{z}$ are unknown non-zero vectors, admits the solutions for the unknown scalars.

PROOF:

The given vector equation is

$$\lambda\vec{x} + \mu\vec{y} = \vec{z} \qquad \textbf{... (1.1)}$$

Taking the cross product of (1) with y we obtain by virtue of

$\vec{y} \times \vec{y} = \vec{0}$ that

$$\lambda\vec{x} \times \vec{y} = \vec{z} \times \vec{y} \ ,$$

Which further yields, on taking dot product with $\vec{z} \times \vec{y}$,

$$\lambda(\vec{x} \times \vec{y}) \cdot (\vec{z} \times \vec{y}) = (\vec{z} \times \vec{y}) \cdot (\vec{z} \times \vec{y})$$

$$\lambda = \frac{|\vec{z}\times\vec{y}|^2}{(\vec{x}\times\vec{y})\cdot(\vec{z}\times\vec{y})},$$

provided that the vectors $\vec{x}$ *and* $\vec{y}$, and also $\vec{y}$ *and* $\vec{z}$ are not parallel to each other. Now if they are parallel then we have $\vec{x} = v\vec{y}$, where v is a scalar. Then **(1.1)** reduces to

$$(\lambda v + \mu)\vec{y} = \vec{z},$$

Which shows that y *and* z are parallel and hence there are infinite number of solutions of λ *and* μ.

Further processing as in above we obtain $\boldsymbol{\mu} = \frac{|\vec{\boldsymbol{z}}\times\vec{\boldsymbol{x}}|^{\mathbf{2}}}{(\vec{\boldsymbol{y}}\times\vec{\boldsymbol{x}})\cdot(\vec{\boldsymbol{z}}\times\vec{\boldsymbol{x}})}$.

THEOREM: 1.3

The vector equation $\vec{\boldsymbol{r}}.\vec{\boldsymbol{p}} = \boldsymbol{\lambda}$, where λ is a known scalar and $\vec{p}$ is a non-zero known vector, admits a solution.

PROOF:

The given vector equation can be written as

$$\vec{r}.\vec{p} = \lambda \frac{\vec{p}}{|\vec{p}|^2} \cdot \vec{p}$$

$$\left(\vec{r} - \lambda \frac{\vec{p}}{|\vec{p}|^2}\right) \cdot \vec{p} = 0,$$

Which implies that the vector $\vec{\boldsymbol{r}} - \boldsymbol{\lambda} \frac{\vec{\boldsymbol{p}}}{|\vec{\boldsymbol{p}}|^2}$ is orthogonal to $\vec{p}$ and hence we have

$$\vec{r} - \lambda \frac{\vec{p}}{|\vec{p}|^2} = \vec{p} \times \vec{q},$$

Where $\vec{q}$ is a non-zero arbitrary vector. Thus

$r = \lambda \frac{\vec{p}}{|\vec{p}|^2} + (\vec{p} \times \vec{q})$ is the general solution of the given equation.

STRAIGHT LINE

Vector algebra supplies us several indispensable tool by which we can study straight line various conditions.

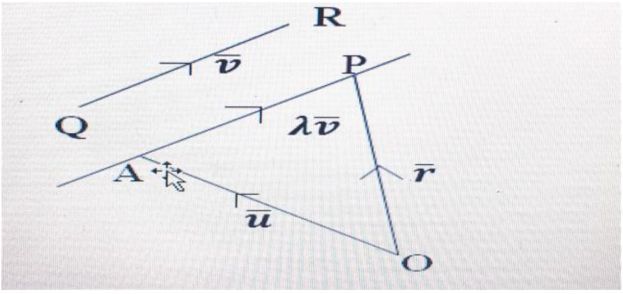
Straight line through a point and parallel to a vector

We consider that A is a given point, $\vec{v}$ be a given vector and P is any point on the straight line whose equation is to be determined. Related to a fixed point O chosen as origin, suppose that $\vec{u}$ and $\vec{r}$ are the positions vectors of A and P respectively.

Then we have $\overrightarrow{AP} = \vec{r} - \vec{u}$. since $\overrightarrow{AP}$ is parallel to $\vec{v}$ so we must have $\overrightarrow{AP} = \lambda\vec{v}, \lambda$ being a scalar quantity, which assumes different values for different positions of the variable point P on the line. Thus from above we obtain

$$\vec{r} - \vec{u} = \lambda\vec{v}, \;\; i.e. \; \vec{r} = \vec{u} + \lambda\vec{v}, \qquad \textbf{....(2.1)}$$

Which is the required vector equation of the straight line.



Corollary 1:

If the straight line passes through the origin, then the equation of the straight line (2.1) reduces to $\vec{r} = \lambda\vec{v}$.

Corollary 2:

If $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}, \vec{u} = u_1\hat{\imath} + u_2\hat{\jmath} + u_3\hat{k}$ and

$\vec{v} = \; v_1\hat{\imath} + v_2\hat{\jmath} + v_3\hat{k},$ then from (2.1) we obtain

$$x\hat{\imath} + y\hat{\jmath} + z\hat{k} = u_1\hat{\imath} + u_2\hat{\jmath} + u_3\hat{k} + \lambda\big(v_1\hat{\imath} + v_2\hat{\jmath} + v_3\hat{k}\big),$$

Which yields

$$\frac{\boldsymbol{x-u_1}}{\boldsymbol{v_1}} = \frac{\boldsymbol{y-u_2}}{\boldsymbol{v_2}} = \frac{\boldsymbol{z-u_3}}{\boldsymbol{v_3}} = \boldsymbol{\lambda},$$

Which is the equation of the straight line in three dimensional Cartesian coordinate geometry passing through (u_1, u_2, u_3) with direction cosines proportional to (v_1, v_2, v_3).

NOTE:

The point P is one side or the other of the point A, with respect to the vector $\vec{v}$, according as λ is positive or negative; λ is known as the scalar parameter and the equations obtained above are known as the parametric forms.

Corollary 3: (non-parametric form).

From the above discussion it follows that the vectors

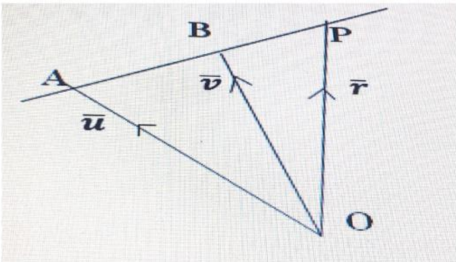
$\vec{r} - \vec{u}\ and\ \vec{v}$ are parallel to each other and hence we obtain

$$(\vec{r} - \vec{u}) \times \vec{v} = \vec{0}, i.e. \vec{r} \times \vec{u} = \vec{r} \times \vec{v}, \qquad \textbf{... (1.2)}$$

Which is the equation of the straight line in non-parametric form.

Straight Line Through Two Points

We consider that A and B are two given points. Relative to a fixed point O chosen as origin, suppose that $\vec{u}$ and $\vec{v}$ are the position vectors of A and B respectively. Then



We have $\overrightarrow{AB} = \vec{v} - \vec{u}$. Now let P be any point with position vector $\vec{r}$ on the straight line whose equation is to be determined. Thus we are going to find the equation of a line which passes through $\vec{u}$ and parallel to the vector $\vec{v} - \vec{u}$. hence proceeding as in the previous subsection we obtain the equation of the line as follows:

$\vec{r} - \vec{u} = \lambda(\vec{v} - \vec{u})$, i.e. $\vec{r} = (1 - \lambda)\vec{u} + \lambda\vec{v},$ **...... (2.3)**

Where λ is a scalar quantity.

CONCLUSION

The vector algebra play fundamental role in the straight line, plane and sphere. In this applications to various fields such as geometry, mechanics, physics, and engineering, scientific research. The concept of vector algebra is explained lucidly with the geometric notations and physical motivations. Further the geometrical and physical applications through this chapter help one in the advanced reading of the subject which is becoming more and more abstract.

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