

TRAPEZOIDAL NEUTROSOPHIC FUZZY GRAPHS

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Abstract

The concept of this research is introduced to interval-valued trapezoidal neutrosophic fuzzy graph which is combined to trapezoidal fuzzy numbers and interval-valued neutrosophic fuzzy graph. In this analysis, proposed algorithm finds source node and destination node because of the shortest path problem. In this research, we apply trapezoidal number with interval-valued neutrosophic fuzzy graph and finding their score function. Eventually an illustrative example to explain, to easy way of shortest path fuzzy graph.

Keywords

Score function (SF), Shortest Path Problem (SPP), Interval-valued Trapezoidal Number.

I. INTRODUCTION

Shortest path problems are very helpful for finding short way of transportation problem. This types of problem benefits are time saving and is less stressful. In this research we introduced trapezoidal number with interval-valued neutrosophic fuzzy graph. We calculate the shortest path using above format.

The SPP of a network graph, whose edge distance is assume and calculated interval-valued trapezoidal neutrosophic number. Ye and Peng et al. used score function and trapezoidal fuzzy neutrosophic number [5,13]. Broumi et al. proffer shortest path problem nether interval-valued neutrosophic setting [27].

In this paper, we conclude shortest path using interval-valued trapezoidal neutrosophic fuzzy graph through proposed algorithm.

II. METHODOLOGY

In this section we explain some important definition.

Definition 2.1 [10]

Let $\bar{n}_1 = \langle [(t_a^L, t_b^L, t_c^L, t_d^L), (t_a^U, t_b^U, t_c^U, t_d^U)], [(i_a^L, i_b^L, i_c^L, i_d^L), (i_a^U, i_b^U, i_c^U, i_d^U)], [(f_a^L, f_b^L, f_c^L, f_d^L), (f_a^U, f_b^U, f_c^U, f_d^U)] \rangle$ and $\bar{n}_2 = \langle [(T_a^L, T_b^L, T_c^L, T_d^L), (T_a^U, T_b^U, T_c^U, T_d^U)], [(I_a^L, I_b^L, I_c^L, I_d^L), (I_a^U, I_b^U, I_c^U, I_d^U)], [(F_a^L, F_b^L, F_c^L, F_d^L), (F_a^U, F_b^U, F_c^U, F_d^U)] \rangle$ both interval-valued trapezoidal neutrosophic numbers. Therefore following procedure are hold :

$$(1) \bar{n}_1 \oplus \bar{n}_2 = \langle [(t_a^L + T_a^L - t_a^L T_a^L, t_b^L + T_b^L - t_b^L T_b^L, t_c^L + T_c^L - t_c^L T_c^L, t_d^L + T_d^L - t_d^L T_d^L), (t_a^U + T_a^U - t_a^U T_a^U, t_b^U + T_b^U - t_b^U T_b^U, t_c^U + T_c^U - t_c^U T_c^U, t_d^U + T_d^U - t_d^U T_d^U)], [(i_a^L I_a^L, i_b^L I_b^L, i_c^L I_c^L, i_d^L I_d^L), (i_a^U I_a^U, i_b^U I_b^U, i_c^U I_c^U, i_d^U I_d^U)], [(f_a^L F_a^L, f_b^L F_b^L, f_c^L F_c^L, f_d^L F_d^L), (f_a^U F_a^U, f_b^U F_b^U, f_c^U F_c^U, f_d^U F_d^U)] \rangle$$

We propose definition of score and accuracy functions for an interval-valued trapezoidal neutrosophic number.

Definition 2.2 [10]

Let $\bar{n} = \langle [(t_a^L, t_b^L, t_c^L, t_d^L), (t_a^U, t_b^U, t_c^U, t_d^U)], [(i_a^L, i_b^L, i_c^L, i_d^L), (i_a^U, i_b^U, i_c^U, i_d^U)], [(f_a^L, f_b^L, f_c^L, f_d^L), (f_a^U, f_b^U, f_c^U, f_d^U)] \rangle$ be an interval-valued trapezoidal neutrosophic number, then defined as their score functions

$$S(\bar{n}) = \left(\frac{t_a^U + t_b^U + t_c^U + t_d^U}{4} + \frac{t_a^L + t_b^L + t_c^L + t_d^L}{4} \right) + \left(\frac{i_a^U + i_b^U + i_c^U + i_d^U}{4} - \frac{i_a^L + i_b^L + i_c^L + i_d^L}{4} \right) + \left(\frac{f_a^U + f_b^U + f_c^U + f_d^U}{4} - \frac{f_a^L + f_b^L + f_c^L + f_d^L}{4} \right), S(\bar{n}) \in [-1, 1], \dots \quad (1)$$

Where the higher value of $S(\bar{n})$, larger the interval-valued trapezoidal number \bar{n} .

NETWORK TERMINOLOGY :

Consider a network $G(V, E)$, We denotes interval-valued trapezoidal neutrosophic fuzzy graph corresponding with the edge (i, j) , the neutrosophic distance path

$$d(A) = \sum_{(i,j \in A)} d_{ij}$$

III. INTERVAL-VALUED TRAPEZOIDAL NEUTROSOPHIC FUZZY GRAPH

In this research, we using proposed algorithm for finding shortest path.

Step :1

Let $d_1 = \langle [(0, 0, 0, 0), (0, 0, 0, 0)], [(1, 1, 1, 1), (1, 1, 1, 1)], [(1, 1, 1, 1), (1, 1, 1, 1)] \rangle$ and the source node as $[d_1 = \langle [(0, 0, 0, 0), (0, 0, 0, 0)], [(1, 1, 1, 1), (1, 1, 1, 1)], [(1, 1, 1, 1), (1, 1, 1, 1)] \rangle]$.

Step: 2

Find $d_j = \text{minimum } \{d_i \oplus d_{ij}\}; j = 2, 3, \dots, n$

Step: 3

If the minimum value of i . ie., $i=r$ then the lable node j as $[d_j, r]$. If minimum arise related to more than one values of i . Their position we choose minimum value of i .

Step: 4

Let the destination node be $[d_n, l]$. Here source node is d_n . We conclude a score function and we find minimum value of interval-valued trapezoidal neutrosophic number.

Step: 5

We calculate shortest path problem between source and destination node. Review the label of node 1. Let it be as $[d_n, A]$. Now review the label of node A and so on. Replicate the same procedure until node 1 is procured.

Step: 6

The shortest path can be procured by combined all the nodes by the step 5.

IV. ILLUSTRATIVE EXAMPLE

Illustrative the above procedure to solve shortest path problem through interval-valued trapezoidal neutrosophic fuzzy graph.

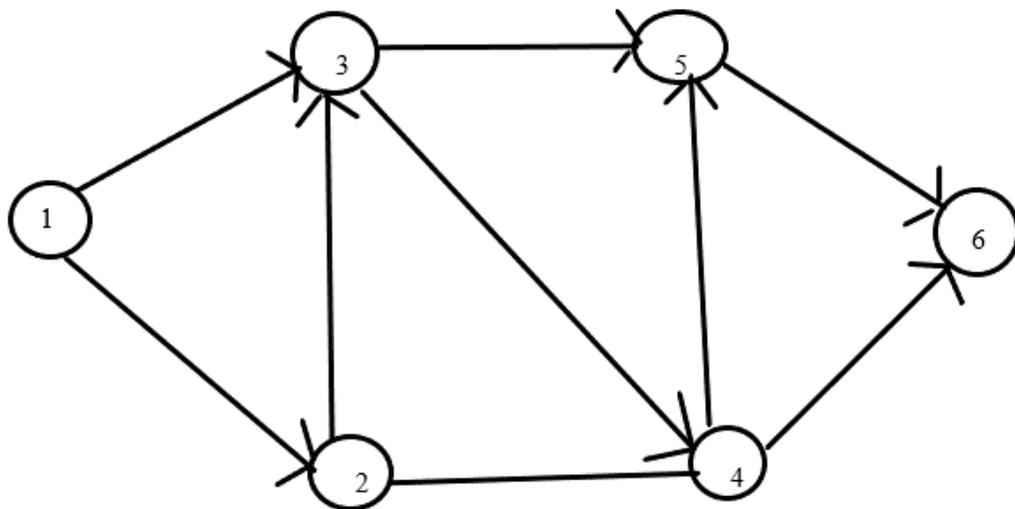


Fig 1: Interval-valued trapezoidal neutrosophic fuzzy graph

Edges	Interval-valued trapezoidal fuzzy neutrosophic numbers
1-2	$\langle [(0.12, 0.25, 0.46, 0.75), (0.46, 0.37, 0.18, 0.25)], [(0.91, 0.25, 0.16, 0.02), (0.25, 0.36, 0.47, 0.58)], [(0.31, 0.42, 0.53, 0.64), (0.49, 0.58, 0.67, 0.52)] \rangle$
1-3	$\langle [(0.25, 0.31, 0.67, 0.81), (0.21, 0.92, 0.34, 0.57)], [(0.42, 0.10, 0.03, 0.21), (0.01, 0.18, 0.05, 0.18)], [(0.61, 0.07, 0.15, 0.21) , (0.75, 0.18, 0.09, 0.15)] \rangle$
2-3	$\langle [(0.25, 0.39, 0.27, 0.91), (0.71, 0.25, 0.39, 0.01)], [(0.04, 0.12, 0.31, 0.42), (0.69, 0.81, 0.71, 0.11)], [(0.59, 0.37, 0.71, 0.92), (0.06, 0.13, 0.29, 0.19)] \rangle$
2-4	$\langle [(0.71, 0.02, 0.15, 0.29), (0.04, 0.15, 0.71, 0.15)], [(0.35, 0.27, 0.19, 0.09), (0.39, 0.27, 0.15, 0.70)], [(0.15, 0.07, 0.95, 0.75), (0.15, 0.25, 0.17, 0.03)] \rangle$
3-4	$\langle [(0.15, 0.07, 0.19, 0.29), (0.33, 0.44, 0.19, 0.07)], [(0.45, 0.22, 0.11, 0.06), (0.66, 0.75, 0.01, 0.52)], [(0.51, 0.43, 0.32, 0.91), (0.57, 0.19, 0.07, 0.29)] \rangle$
3-5	$\langle [(0.75, 0.61, 0.10, 0.72), (0.37, 0.49, 0.29, 0.19)], [(0.09, 0.15, 0.32, 0.42), (0.19, 0.09, 0.34, 0.53)], [(0.49, 0.36, 0.71, 0.82), (0.15, 0.05, 0.19, 0.15)] \rangle$
4-5	$\langle [(0.15, 0.33, 0.49, 0.62), (0.66, 0.17, 0.35, 0.19)], [(0.44, 0.59, 0.63, 0.71), (0.05, 0.19, 0.24, 0.39)], [(0.15, 0.29, 0.37, 0.49), (0.06, 0.29, 0.32, 0.49)] \rangle$
4-6	$\langle [(0.71, 0.69, 0.33, 0.19), (0.07, 0.13, 0.23, 0.19)], [(0.32, 0.19, 0.23, 0.45), (0.08, 0.19, 0.49, 0.57)], [(0.49, 0.67, 0.71, 0.81), (0.29, 0.02, 0.16, 0.26)] \rangle$
5-6	$\langle [(0.15, 0.72, 0.59, 0.32), (0.72, 0.09, 0.25, 0.19)], [(0.25, 0.33, 0.22, 0.07), (0.19, 0.25, 0.35, 0.29)], [(0.71, 0.39, 0.47, 0.37), (0.09, 0.19, 0.25, 0.71)] \rangle$

Table 1: Interval-valued trapezoidal fuzzy neutrosophic edge weight

Let n=6 is the destination node,

$d_1 = \langle [(0, 0, 0, 0), (0, 0, 0, 0)], [(1, 1, 1, 1), (1, 1, 1, 1)], [(1, 1, 1, 1), (1, 1, 1, 1)] \rangle$ and label of source node is $\{ \langle [(0, 0, 0, 0), (0, 0, 0, 0)], [(1, 1, 1, 1), (1, 1, 1, 1)], [(1, 1, 1, 1), (1, 1, 1, 1)] \rangle, \dots \}$ the value of $d_j, j= 2, 3, 4, 5, 6$ is succeeding:

Iteration 1:

Here, node 1 is the forerunner node of node 2. So fix $i=1$ and $j=2$ we apply step 2

$$d_2 = \text{minimum } \{ d_1 \oplus d_{12} \}$$

$$= \text{minimum } \{ \langle [(0, 0, 0, 0), (0, 0, 0, 0)], [(1, 1, 1, 1), (1, 1, 1, 1)], [(1, 1, 1, 1), (1, 1, 1, 1)] \rangle \oplus \langle [(0.12, 0.25, 0.46, 0.75), (0.46, 0.37, 0.18, 0.25)], [(0.91, 0.25, 0.16, 0.02), (0.25, 0.36, 0.47, 0.58)], [(0.31, 0.42, 0.53, 0.64), (0.49, 0.58, 0.67, 0.52)] \rangle \}$$

$$= \{ \langle [(0.12, 0.25, 0.46, 0.75), (0.46, 0.37, 0.18, 0.25)], [(0.91, 0.25, 0.16, 0.02), (0.25, 0.36, 0.47, 0.58)], [(0.31, 0.42, 0.53, 0.64), (0.49, 0.58, 0.67, 0.52)] \rangle \}$$

Therefore minimum value $i=1$, corresponding to label node 2 as

$$\{ \langle [(0.12, 0.25, 0.46, 0.75), (0.46, 0.37, 0.18, 0.25)], [(0.91, 0.25, 0.16, 0.02), (0.25, 0.36, 0.47, 0.58)], [(0.31, 0.42, 0.53, 0.64), (0.49, 0.58, 0.67, 0.52)] \rangle, 1 \}$$

$$d_2 = \{ \langle [(0.12, 0.25, 0.46, 0.75), (0.46, 0.37, 0.18, 0.25)], [(0.91, 0.25, 0.16, 0.02), (0.25, 0.36, 0.47, 0.58)], [(0.31, 0.42, 0.53, 0.64), (0.49, 0.58, 0.67, 0.52)] \rangle \}$$

Iteration 2:

Here forerunner node of node 3 are nodes 1 and 2. So fix $i=1,2$ and $j=3$ we apply step 2 .

$$d_3 = \text{minimum } \{ d_1 \oplus d_{13}, d_2 \oplus d_{23} \}$$

$$= \text{minimum } \{ \langle [(0, 0, 0, 0), (0, 0, 0, 0)], [(1, 1, 1, 1), (1, 1, 1, 1)], [(1, 1, 1, 1), (1, 1, 1, 1)] \rangle \oplus \dots \}$$

$[(0.25, 0.31, 0.67, 0.81), (0.21, 0.92, 0.34, 0.57)], [(0.42, 0.10, 0.03, 0.21), (0.01, 0.18, 0.05, 0.18)], [(0.61, 0.07, 0.15, 0.21), (0.75, 0.18, 0.09, 0.15)] >, < [(0.12, 0.25, 0.46, 0.75), (0.46, 0.37, 0.18, 0.25)], [(0.91, 0.25, 0.16, 0.02), (0.25, 0.36, 0.47, 0.58)], [(0.31, 0.42, 0.53, 0.64), (0.49, 0.58, 0.67, 0.52)] \oplus [(0.25, 0.39, 0.27, 0.91), (0.71, 0.25, 0.39, 0.01)], [(0.04, 0.12, 0.31, 0.42), (0.69, 0.81, 0.71, 0.11)], [(0.59, 0.37, 0.71, 0.92), (0.06, 0.13, 0.29, 0.19)] >\}$

$= \text{minimum} \{ < [(0.25, 0.31, 0.67, 0.81), (0.21, 0.92, 0.34, 0.57)], [(0.42, 0.10, 0.03, 0.21), (0.01, 0.18, 0.05, 0.18)], [(0.61, 0.07, 0.15, 0.21), (0.75, 0.18, 0.09, 0.15)] >, < [(0.34, 0.54, 0.61, 0.98), (0.84, 0.53, 0.50, 0.26)], [(0.04, 0.03, 0.05, 0), (0.17, 0.29, 0.33, 0.06)], [(0.18, 0.16, 0.38, 0.59), (0.03, 0.08, 0.19, 0.10)] >\}$

$S \{ < [(0.25, 0.31, 0.67, 0.81), (0.21, 0.92, 0.34, 0.57)], [(0.42, 0.10, 0.03, 0.21), (0.01, 0.18, 0.05, 0.18)], [(0.61, 0.07, 0.15, 0.21), (0.75, 0.18, 0.09, 0.15)] >\}$

Using equation(1),we have

$$S(\bar{n}_1) = 0.97$$

$S \{ < [(0.34, 0.54, 0.61, 0.98), (0.84, 0.53, 0.50, 0.26)], [(0.04, 0.03, 0.05, 0), (0.17, 0.29, 0.33, 0.06)], [(0.18, 0.16, 0.38, 0.59), (0.03, 0.08, 0.19, 0.10)] >\}$

$$S(\bar{n}_2) = 1.105$$

Therefore minimum value $i=1$, corresponding to label node 3 as

$\{ < [(0.25, 0.31, 0.67, 0.81), (0.21, 0.92, 0.34, 0.57)], [(0.42, 0.10, 0.03, 0.21), (0.01, 0.18, 0.05, 0.18)], [(0.61, 0.07, 0.15, 0.21), (0.75, 0.18, 0.09, 0.15)] >, 1 \}$

$d_3 = \{ < [(0.25, 0.31, 0.67, 0.81), (0.21, 0.92, 0.34, 0.57)], [(0.42, 0.10, 0.03, 0.21), (0.01, 0.18, 0.05, 0.18)], [(0.61, 0.07, 0.15, 0.21), (0.75, 0.18, 0.09, 0.15)] >\}$

Iteration 3:

The forerunner node of node 4 are nodes 2 and 3. So fix $i=2,3$ and $j=4$ we apply step 2 .

$$d_4 = \text{minimum} \{ d_2 \oplus d_{24}, d_3 \oplus d_{34} \}$$

Therefore,

$$S(\bar{n}_1) = 0.99$$

$$S(\bar{n}_2) = 1.18$$

Therefore minimum value $i=2$, corresponding to lable node 4 as

$$\{ < [(0.74, 0.27, 0.54, 0.82), (0.48, 0.46, 0.76, 0.36)], [(0.32, 0.07, 0.03, 0), (0.10, 0.10, 0.07, 0.40)], [(0.05, 0.03, 0.50, 0.48), (0.07, 0.15, 0.11, 0.02)] >, 2 \}$$

$$\therefore d_4 = \{ < [(0.74, 0.27, 0.54, 0.82), (0.48, 0.46, 0.76, 0.36)], [(0.32, 0.07, 0.03, 0), (0.10, 0.10, 0.07, 0.40)], [(0.05, 0.03, 0.50, 0.48), (0.07, 0.15, 0.11, 0.02)] > \}$$

Iteration 4:

The forerunner node of node 5 are nodes 3 and 4. So fix $i=3,4$ and $j=5$ we apply step 2 .

$$d_5 = \text{minimum} \{ d_3 \oplus d_{35}, d_4 \oplus d_{45} \}$$

Therefore,

$$S(\bar{n}_1) = 1.43$$

$$S(\bar{n}_2) = 1.07$$

Therefore minimum value $i=4$, corresponding to lable node 5 as

$$\{ < [(0.78, 0.51, 0.77, 0.93), (0.82, 0.55, 0.84, 0.48)], [(0.14, 0.04, 0.02, 0), (0.01, 0.02, 0.02, 0.16)], [(0.01, 0.01, 0.19, 0.23), (0, 0.04, 0.04, 0.01)] >, 4 \}$$

$$d_5 = < [(0.78, 0.51, 0.77, 0.93), (0.82, 0.55, 0.84, 0.48)], [(0.14, 0.04, 0.02, 0), (0.01, 0.02, 0.02, 0.16)], [(0.01, 0.01, 0.19, 0.23), (0, 0.04, 0.04, 0.01)] >$$

Iteration 5:

The forerunner node of node 6 are nodes 4 and 5. So fix $i=4, 5$ and $j=6$ we apply step 2.

$$d_6 = \text{minimum} \{d_4 \oplus d_{46}, d_5 \oplus d_{56}\}$$

Therefore,

$$S(\bar{n}_1) = 1.26$$

$$S(\bar{n}_2) = 1.59$$

Therefore minimum value $i=4$, corresponding to label node 6 as

$$\{ < [(0.93, 0.77, 0.69, 0.85), (0.52, 0.53, 0.82, 0.48)], [(0.10, 0.01, 0.01, 0), (0.01, 0.02, 0.03, 0.23)], [(0.02, 0.02, 0.36, 0.39), (0.02, 0, 0.02, 0.01)] >, 4 \}$$

$$d_6 = < [(0.93, 0.77, 0.69, 0.85), (0.52, 0.53, 0.82, 0.48)], [(0.10, 0.01, 0.01, 0), (0.01, 0.02, 0.03, 0.23)], [(0.02, 0.02, 0.36, 0.39), (0.02, 0, 0.02, 0.01)] >$$

Here, the shortest path between node 1 and node 6 is

$$< [(0.93, 0.77, 0.69, 0.85), (0.52, 0.53, 0.82, 0.48)], [(0.10, 0.01, 0.01, 0), (0.01, 0.02, 0.03, 0.23)], [(0.02, 0.02, 0.36, 0.39), (0.02, 0, 0.02, 0.01)] >$$

To find shortest path :

Already we know node 6 is labeled by

$$\{ < [(0.93, 0.77, 0.69, 0.85), (0.52, 0.53, 0.82, 0.48)], [(0.10, 0.01, 0.01, 0), (0.01, 0.02, 0.03, 0.23)], [(0.02, 0.02, 0.36, 0.39), (0.02, 0, 0.02, 0.01)] >, 4 \}$$

And , node 4 is labeled by

$$\{ < [(0.74, 0.27, 0.54, 0.82), (0.48, 0.46, 0.76, 0.36)], [(0.32, 0.07, 0.03, 0), (0.10, 0.10, 0.07, 0.40)], [(0.05, 0.03, 0.50, 0.48), (0.07, 0.15, 0.11, 0.02)] >, 2 \}$$

Node 2 is labeled by

$$\{ < [(0.12, 0.25, 0.46, 0.75), (0.46, 0.37, 0.18, 0.25)], [(0.91, 0.25, 0.16, 0.02), (0.25, 0.36, 0.47, 0.58)], [(0.31, 0.42, 0.53, 0.64), (0.49, 0.58, 0.67, 0.52)] >, 1 \}$$

First we take source node , node 1. Then we combine all the obtained interval-valued trapezoidal neutrosophic nodes. Finally we get the shortest path is

$$1 \rightarrow 2 \rightarrow 4 \rightarrow 6.$$

Node	d_i	Interval-valued trapezoidal fuzzy neutrosophic shortest path between j^{th} and 1^{st} node
2	[(0.12, 0.25, 0.46, 0.75), (0.46, 0.37, 0.18, 0.25)], [(0.91, 0.25, 0.16, 0.02), (0.25, 0.36, 0.47, 0.58)], [(0.31, 0.42, 0.53, 0.64), (0.49, 0.58, 0.67, 0.52)]	1 → 2
3	[(0.25, 0.31, 0.67, 0.81), (0.21, 0.92, 0.34, 0.57)], [(0.42, 0.10, 0.03, 0.21), (0.01, 0.18, 0.05, 0.18)], [(0.61, 0.07, 0.15, 0.21), (0.75, 0.18, 0.09, 0.15)]	1 → 3
4	[(0.74, 0.27, 0.54, 0.82), (0.48, 0.46, 0.76, 0.36)], [(0.32, 0.07, 0.03, 0), (0.10, 0.10, 0.07, 0.40)], [(0.05, 0.03, 0.50, 0.48), (0.07, 0.15, 0.11, 0.02)]	1 → 2 → 4
5	[(0.78, 0.51, 0.77, 0.93), (0.82, 0.55, 0.84, 0.48)], [(0.14, 0.04, 0.02, 0), (0.01, 0.02, 0.02, 0.16)], [(0.01, 0.01, 0.19, 0.23), (0, 0.04, 0.04, 0.01)]	1 → 2 → 4 → 5
6	[(0.93, 0.77, 0.69, 0.85), (0.52, 0.53, 0.82, 0.48)], [(0.10, 0.01, 0.01, 0), (0.01, 0.02, 0.03, 0.23)], [(0.02, 0.02, 0.36, 0.39), (0.02, 0, 0.02, 0.01)]	1 → 2 → 4 → 6

Table 2: Interval-valued trapezoidal fuzzy neutrosophic distance and shortest path

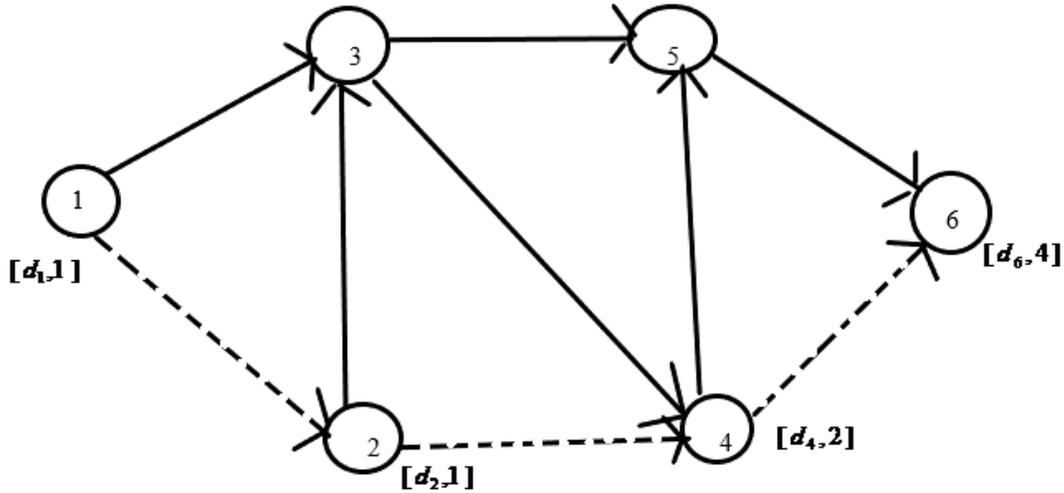


Fig 2 : Interval-valued trapezoidal fuzzy neutrosophic distance from node 1.

V CONCLUSION

In this research, we have apply an proposed algorithm for solving the interval-valued trapezoidal neutrosophic fuzzy graph. we find all possible path from source node to destination node through interval-valued trapezoidal number. We find rank of the path based on score function. The path corresponding to lowest rank is specified to shortest path. Eventually corresponding to all the nodes combined, we get the shortest path.

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