THE DOMINATION OF ZERO DIVISOR GRAPH ON THE LINE GRAPH

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1.ABSTRACT:

The directed Zero divisor graph is a graph constructed out of a non-Commutative ring R and its non-zero divisors. In this paper we find various domination parameter for the zero-divisor graph. For all commutative rings provides characterization with unity of total perfect codes. In this paper we study the characterization of eigenvaluesand adjacency matrix. We find the numerical example of non-zero divisor graph.

2.KEYWORDS:

Ring, Zero divisor, Zero-divisor graph, Zero divisor on a matrix.

3.INTRODUCTION:

Here two variances of graph using zero-divisor consistently apply. Theory of perfect Zero-divisors graphs firms anrings interesting pass of graph theory and has connections in group theory, number theory and cryptography. It acted a central role in the fast growing of emr-geometry ring theory. Atbari and Muhammadian improved many renules in ring Zero divisors graphs.

Here, domination graph are obtained. Let zero divisor R be ring with identityand set of all nonzero. Units of R are group of all g and nonzero non units of R is Y

Let g be the group of all units of R and Y is the set of all nonzero, zero divisor of R. The group action on Y by g is given by $(y_1 g_1) \to y g_1$

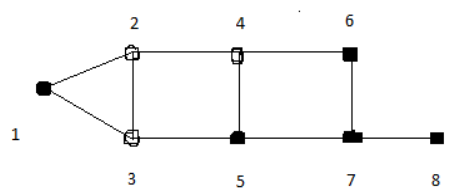
form Y×G to Y is called proper regular action then for each$y_1 \in Y$,

$P_r(y_1) = \{\varphi(y_1, g) = y_1 g : \forall g \in G\}$is called right orbit of y_1. If $Z(R)^* = Y$ then$Z(R)^*$is called finite ring. Let q be a prime number and $= M_2(X_q)$. Then for any M$\epsilon L(R^*)$ the number of orbits under the right regular action on Z(R^*) by g is q-1 and the number of nonzero nilpotents in R is $q^2 + 1$.

4.DOMINATIONS(ZERO DIVISOR GRAPHS) :

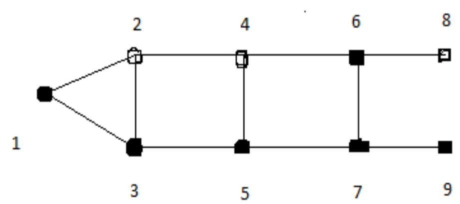
The result of this division provide effective criterion for discussing the dominating sets and the connected dominating sets in zero divisor graph of $M_2(Xq)$ observed. A dominating set$g = (v, e)$ is a subset $\mathcal{D}$ of $\mathcal{V}$ is every vertex not in $\mathcal{D}$ such that adjoining to at the minimumone member of $\mathcal{D}$. The domination numbers $\beta(g)$ is the number of vertices in a smallest dominating set for g.

4.1-EXAMPLE:



Dominating sets

$D_1 = \{1,5,7,6,8\}$

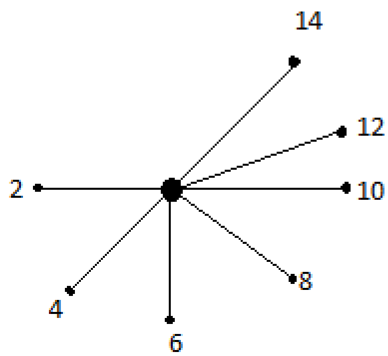


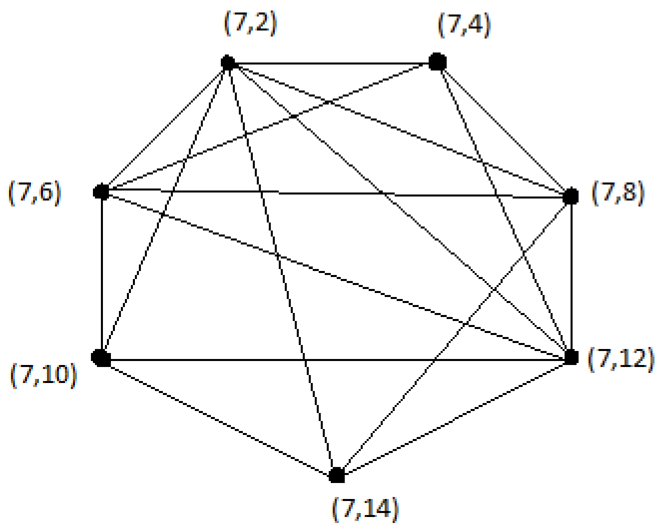
$D_2 = \{3,5,7,6\}$

5.DOMINATION NUMBER (ZERO DIVISOR GRAPHS):

5.1-EXAMPLE:

$\Gamma(Z16) = \{2,4,6,7,8,10,12,14\}$





5.2-THEOREM:

For ΓZ_m if m = 2q where q, q is odd number q>3 then the domination number of zero divisor is one.

PROOF:

If for m = 2q

hereΓZ_m is a graph.

Therefore common vertex is adjacent to any other vertices.

Draw the zero divisor ΓZ_m. Now m = 2q and Let w_1 be the ordinary peakof ΓZ_m , Every edges end vertex is ΓZ_m.

Now$w_1$appears in each vertex of zero divisor is$[w_i, u_i]\epsilon L(\Gamma Z_m)$.

Now fix the point$[w_i, u_i]$ which is adjoining to recurrentvertices L(ΓZ_m). It'sdesign isfull graphs. The domination number of a zero divisor graph is one.

Therefore, m = 2q.

5.3-EXAMPLE:

The complete graph is integer modulo m of the ring of the zero divisor graph or complete Bipartite graph.

Therefore zero divisor of $Z_3 \times Z_6$ the only possible zero divisor graph that is tree but not a star.

5.4-THEOREM:

The zero divisor domination graph of the form ΓZ_m if

n = 5q , q is an different prime number q > 2 then the domination sum =$\{2\ if\ q < 6, 4\ if\ q \geq 6$

PROOF:

CASE 1:

Let q < 6 vertices in two independent set is ΓZ_mThen its adjacent to all vertices .Let zero divisor ΓZ_mafter that q + 4 and q + 2 are one set and another set vertices in L(ΓZ_m)

Assume $[u_{11}, v_{11}]$& $[u_{22}, v_{22}] \epsilon L(\Gamma Z_m)$ which are not adjacent to each other but $[u_{11}, v_{11}]$adjacent $[u_{22}, v_{22}] \epsilon L(\Gamma Z_m)$enduring the raised process we find that these vertices are associated and $v[(Z_n)]$ is associated. Hence they compose dominating set.

So domination number = 2 if q < 6.

CASE 2:

Let m = 6q where q is number of odd prime & q > 6.

PROOF:

$V(\Gamma(Z_M)) = \{6,12,18, \dots \dots .6(q-1), q, 2q, 3q, 4q, 5q\}$

Decompose the vertex set into the following disjoint subsets.

$S_1 = \{6,12,18, \dots \dots \dots 6(q-1)\}$&

$M_1 = \{q, 2q, 3q, 4q, 5q\}$

Now we draw the $\Gamma 2n$ such that

$$V(L(\Gamma Z_m)) = (q,6)(q,12)(q,18) \dots\dots.(q,6(q-1)),$$
$$(2q,6)(2q,12) \dots\dots..(2q,6(q-1)),$$
$$(3q,6)(3q,12) \dots\dots..(3q,6(q-1)),$$
$$(4q,6)(4q,12) \dots\dots\dots(4q,6(q-1)),$$
$$(5q,6)(5q,12) \dots\dots\dots(5q,6(q-1))$$

Decompose these vertex into disjoint subsets

$$K_1 = \{(q,6),(q,12),(q,18),\dots\dots\dots,(q,6(q-1))\}$$

$$L_1 = \{(2q,6),(2q,12),(2q,18)\dots\dots,(2q,6(q-1))\}$$

$$M_1 = \{(3q,6),(3q,12),(3q,18),\dots\dots,(3q,6(q-1))\}$$

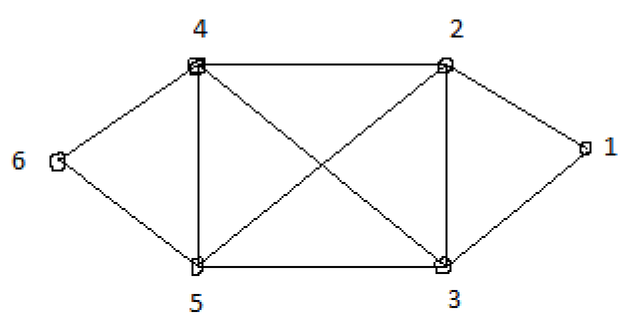
$$O_1 = \{(4q,6),(4q,12),(4q,18),\dots\dots.,(4q,6(q-1))\}$$

Let us accept and introduce with any one of the vertices reciprocal to (L(ΓZ_m))

In this$\{(q,6),(2q,6),(3q,6),(4q,6),(5q,6)\}$ are adjacent and combined. Moreover its correlative is third combined.

Nowdecisioncontroldevelop the dominating set. Therefore domination number = 6.

6.TYPES OF ZERO DIVISOR GRAPHS:



Heredistinct types of zero divisor graph. Let r,s,t are three difinite primes and following cases appear. A proper prime dominating number of zero divisor graph contains no smaller primes and the min (ΓZ_m) is denoted minimal prime numbers in zero divisor's. Complets zero divisor dominating graphs, denoted by $K(\Gamma Z_m)$.

6.1-THEOREM:

Let q ,ΓZ_m be prime and Zero-divisor domination graph of $R_1 = M_2(Z_q)$. Then irredundance number of zero divisor domination graph is q + 1.

PROOF:

Let $Z(R_1)^* = \bigcup_{i=1}^{q} O_{l_1}(M_{i1})$ for some $M_0, M_1, \dots \dots \dots, M_p \in Z(R_1)^*$

Let $\xi = \{V_{11}, V_{12}, \dots \dots \dots, V_{q1}\}$ where $V_{i1} \in O_{l_1}(M_{i1})$for $0 \leq i_1 \leq q_1$.

Then $|\xi| = q + 1$.

CLAIM:

$V_i \cap V_j = \emptyset \ \forall \ i \neq j$

Suppose M_{i1} is a nonzero matrix in $V_i \cap V_j$ for some $i \neq j$

Then $V_i OM = V_j M = 0$

$M \in X_r \ \forall \ X \in Ol_1(V_i)$and

$M \in Y \ for \ all \ Y \in Ol_1(V_j)$

Since $O_{l_1}(V_i) \cap O_l(V_j) = \emptyset$

$id(M_1) \geq 2q^2 - 2 > q^2 - 1$acontradiction.

Thus $N^+[V_i] \rightarrow N^+[\xi - V_i] \neq \emptyset \ for \ all \ i, 0 \leq i \leq q$ and so every element in ξ has a private onto neighbours.

Hence ξ is an irredundant set of D_1 and so $D_1 \leq q + 1$

Suppose ξ_1' is an irredundant set of $D_1$with $|\xi_1'| > q + 1$.

Then ξ' contains atleast two elements from any one of the orbit of $Z(R_1)^*$ with candinality greater than q+1 is not an irredundant set and so ξ is both minimum and maximum cardinality of a maximal irredundant set of D_1.

Hence $D_1 = IR(D_1) = q + 1$.

6.2-LEMMA:

Let D_1 be the directed zero divisor graph on $R_1 = M_2(Z_q)$ and Let $Z(R_1)^* = \bigcup_{i=0}^{q} Ol_1(M_j)$ for some $M_1, M_2, \dots \dots, M_q \epsilon Z(R_1)^*$.

Then ξ is an (β^{+}) set of D_1 iff ξ contains exactly one element in $Ol_1(M_j)$ for each j with $0 \leq j \leq q$.

PROOF:

Suppose ξ contains exactly one element from $O_{l_1}(M_j)$ for each j, $0 \leq j \leq q$.

Let $\xi = \{w_1, w_2, \ldots \ldots, w_q\}$ where $w_j \in O_{l_1}(M_j)$for $0 \leq j \leq q$.

Then ξ is an set of D.

Conversely, suppose ξ is an set of D $|\xi| = q + 1$.

Suppose $O_{l_1}\left(M_j\right) \cap \xi = \emptyset$ for some j.

Then ξ contains atleast two vertices β_1, β_2 form $O_{l_1}\left(M_{j1}\right)$ for some $j \neq i$.

Therefore, $B_{11} = B_{12} = M_{j1}$ both $B_{11}\&B_{12}$ have no private onto neighbour a contradiction.

6.3-PROPOSITION:

Let D_1 be the directed domination zero divisor graph on $R_1 = M_2(Z_q)$. Then $\beta_1(D) = \beta_2(D) = \gamma(D) = q + 1$.

PROOF:

$Z(R_1)^* = \cup_{i=1}^{q} Ol_1\left(A_j\right)$

Where $S = \{B_0, B_1, \ldots \ldots . B_q\}$ is a set of nilpotents in R_1.

Let $V_j = Ol_1(A_j)$ for some j.

Then $\left|V_j \cap O_{l_1}(A_j)\right| = q - 1$ and so there exists $V_j \in O_{l_1}(A_j)$ such that $(V_i, V_j) \in B$ for all $j \neq i$.

Let $\Omega = \{W_0, W_1, \ldots \ldots \ldots, W_q\}$ is a β^{+} set of D.

Also the sub diagraph induced by ξ contains no isolated vertices and the underlying graph is connected. Then ξ is a total as well as weakly connected dominating set of D_1 and so $\xi(D_1) = \xi(D_2) = q + 1$

Let $Y_j \in O_{l_1}(B_i) \cap (A_j)$ and $X_j \in A_j \cap O_{l_1}(A_j)$for $i \neq j$.

Then $(Y_i, Y_j) \in B$ and $(Y_j, Y_i) \in B$

There exists $Y_k \in O_{l_1}(A_k)$ such that $(Y_j, Y_k) \in B$ for each k, $k \neq i \neq j$.

$\xi = \{Y_0, Y_1, \dots \dots \dots Y_i, \dots \dots ., Y_j, \dots \dots \dots Y_q\}$is a β^+set of Dominating graph.

Also the sub digraph induced by ξ is connected and by definition ξ is open dominating set D_1.

Therefore, $\xi(O) = q + 1$.

6.4-COROLLARY:

Let $R_1 = M_2(Z_q)$b is domination graph of directed zero divisor D_1. Then $\beta(O) = \frac{n}{\Delta}$ and ξ is comprise no nilpotent elements andopen dominating set of D_1.

PROOF:

Let ξ is open dominating set of D_1

Suppose ξ comprise a nilpotent element say M_1. Then by lemma $M_1^2 = 0$.

Clearly $M_1 \in O_{l_1}(M_j)$ for some j.

By Lemma $O_{l_1}(M_j) = O_l(M_j) = M$ and so $(D_1 M_1) \notin \xi$ for all $D \in \xi - M$.

Which is a contradiction.

6.5-THEOREM:

Let $D_1$is zero divisor graph of $R_1 = M_2(Z_q)$. If ξ be a minimal dominating set of D_1then ξ is independent iff $B^2 = 0 \ \forall B \in \xi$.

PROOF:

Suppose ξ is an independent dominating set of D_1.

Then $\Omega = \{\beta_0, \beta_1, \dots \dots, \beta_p : B_i \in O_{l_1}(M_j)\ for\ 0 \leq j \leq q$

Suppose $A_j^2 \neq 0$ for some j.

Without loss of generality one can take that $A_j^2 \neq 0$ for some j &$A_j^2 \neq 0$ for some $j \neq i$.

From this we have $A_j = q^2 - 2$&$A_j = q^2 - 1$

Since ξ is independent.

$A_k \notin A_t\ for\ all\ k, t\ \&\ k \neq t$

Now, $|[N^+(\xi)]| = \sum_{l=0,l=i}^{q}|N^+[A_i]| + |N^+[A_i]|$

$$= (q^2 - 1)(q + 1) + 1 > |V_1(D_1)|$$

Which is a contradiction.

Conversely, Suppose $B^2 = 0 \; \forall \, B \in \xi$

Since ξ is a minimal dominating set and by

$\Omega = \{Y_i, Y_i \in O_{l_1}(M_j) \text{and} 0 \leq j \leq q\}$

$X_j \notin D(X_j) \;\; \forall \;\; i \neq j$

The sub graph incited by ξ has no arcs and so ξ has independent dominating set of D.

7.CONCLUSION:

In this article, we explainedingrained domination and evloution in $2[\Gamma Z_m]$. Additionally we discussed theorems and propositions and lemma of zero divisors of dominations. In future this isprotracted ofzero divisor graphs to central graph and total graphs .

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