On Soft Slightly A_RS Continuous Function in Soft Topological Spaces

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Abstract

We introduce Soft A_RS Closed on Soft topological Spaces and study some of their properties. We also investigate the concepts of Soft slightly A_RS continuous functions

Introduction

The soft set theory is a rapidly processing field of mathematics. Molodtsov [7] shown several applications of this theory in solving many practical problems in economics, engineering, social science, medical science, and so on. In 2010 Muhammad shabir, Munazza Naz [8] used soft sets to define a topology namely Soft topology. Soft generalized closed set was introduced by K.Kannan [4] in 2012. The investigation of generalized closed sets has led to several new and interesting concepts like new covering properties and new separation axioms. Some of these separation axioms have been found to be useful in computer science and digital topology. In this paper we defined soft A_RS - closed mapping, soft A_RS - open mapping and a detailed study of some of its properties in soft topological spaces. With the help of counter examples, we show that the noncoincidence of these various types of mappings.

Definition:

Let τ be a collection of soft sets over X with the fixed set E of parameters. Then τ is called a Soft Topologyon X if

i. $\widetilde{\emptyset}, \tilde{X}$ belongs to τ.
ii. The union of any number of soft sets in τ belongs to τ.
iii. The intersection of any two soft sets in τ belongs to τ.

The triplet (X, τ, E) is called Soft Topological Spacesover X.

The members of τ are called Soft Opensets in X and complements of them are called Soft Closedsets in X.

Definition:

Let (X,,E) be a Soft Topological Space over X. Then the subset (A,E) of the Soft topological space (X,τ,E) is called

1. Soft Semi-closed set[2] if int(Cl(A,E)) $\tilde{\subseteq}$ (A,E).
2. Soft generalized Closed set(briefly Soft g-Closed)[8] if Cl(A,E)$\tilde{\subseteq}$ (U ,E) whenever (A,E)$\tilde{\subseteq}$ (U,E) and (U,E) is Soft Open in (X,τ,E).
3. Soft generalized semi Closed set (briefly Soft gs-Closed)[7]if S Cl(A,E) $\tilde{\subseteq}$ (U ,E) whenever (A,E) $\tilde{\subseteq}$ (U,E) and (U,E) is Soft Open in (X,τ,E).

Definition:

Let (X, τ, E) be a soft topological space. A Soft set (F, E) is called soft $A_R S$ - Closed set if βcl(F, E) $\tilde{\subseteq}$ Int (U, E) whenever (F, E) $\tilde{\subseteq}$(U, E) and (U, E) is soft ω - open. The set of all soft $A_R S$ - closed sets is denoted by $A_R S$ C(X).

The respective complements of the above sets are their open forms.

Definition:

A map f: (X,τ,E) → (Y,σ,K) is said to be

1. Soft continuous [3] if inverse image of every Soft open set in (Y,,K) is Soft open in(X,τ,E)

2. Soft semi continuous [3]if inverse image of every Soft open set in (Y,σ,K) is Soft semi open in (X,τ,E)

3.Soft α continuous [4] if inverse image of every Soft open set in (Y,σ,K) is Soft α open in(X,τ,E)

4. Soft generalized semi (gs) continuous [7] if inverse image of every Soft open set in (Y,,K) is Soft gs open in(X,τ,E).

Soft Slightly $A_R S$ Continuous

In this section we exhibit the concept of Soft Slightly A_RS continuous functions and study some of its properties in Soft topological spaces.

Example:

Let X= $\{x_1, x_2\}$, Y = $\{y_1, y_2\}$ and E= $\{e_1, e_2\}$, K = $\{k_1, k_2\}$ and $f : (X,\tau, E) \rightarrow (Y,\sigma,K)$ where $\tau = \{ F_1, F_{13}, F_{15}, F_{16}\}$, $\tau^c = \{ F_{14}, F_2, F_{15}, F_{16}\}$,$SA_RSO(X, \tau, E) = \{ F_{14}, F_{13}, F_6, F_{12}, F_{11}, F_3, F_8, F_7, F_5, F_4, F_1, F_{15}, F_{16}\}$ and $\sigma = \{ F_3, F_{11}, F_{15}, F_{16}\}$, $\sigma^c = \{ F_6, F_5, F_{15}, F_{16}\}$ is defined as $f(F_1) = F_1$, $f(F_2) = F_2$, $f(F_3) = F_3$, $f(F_4) = F_4$, $f(F_5) = F_5$, $f(F_6) = F_6$, $f(F_7) = F_7$, $f(F_8) = F_8$, $f(F_9) = F_9$, $f(F_{10}) = F_{10}$, $f(F_{11}) = F_{11}$, $f(F_{12}) = F_{12}$, $f(F_{13}) = F_{13}$, $f(F_{14}) = F_{14}$, $f(F_{15}) = F_{15}$, $f(F_{16}) = F_{16}$. Clearly f is Soft Slightly A_RScontinuous.

Theorem:

For a function $f: (X,\tau,E) \rightarrow (Y,\sigma,K)$, the following statements are equivalent.

(i) f is Soft slightly A_RS continuous.

(ii) The inverse image of every Soft clopen set (A,E) of Y is Soft A_RSopen in X.

(iii) The inverse image of every Soft clopen set (A,E) of Y is Soft A_RSclosed in X.

(iv) The inverse image of every Soft clopen set (A,E) of Y is Soft A_RSclopen in X.

Proof:

(i) $\Rightarrow$ (ii): Follows from the definition

(ii) $\Rightarrow$ (iii): Let (A,E) be a Soft clopen set in Y which implies $(A,E)^c$ is Soft clopen in Y. By (ii), $f^{-1}((A,E)^c) = (f^{-1}((A,E)))^c$ is Soft A_RSopen in X. Therefore, $f^{-1}((A,E))$ is Soft A_RSclosed in X.

Theorem:

Every Soft slightly continuous function is Soft slightly A_RS continuous.

Proof: Let $f : (X,\tau,E) \rightarrow (Y,\sigma,K)$be a Soft slightly continuous function. If (A,E) be a Soft clopen set in Y. If $f^{-1}((A,E))$ is Soft open in X. Since, every Soft open set is Soft A_RSopen. Hence, f is Soft slightly A_RS continuous .

Remark:

The converse of the above theorem need not be true as can be seen from the following example.

Example:

Let X= $\{x_1, x_2\}$, Y = $\{y_1, y_2\}$ and E= $\{e_1, e_2\}$, K =$\{k_1, k_2\}$ and $f : (X,\tau, E) \rightarrow (Y,\sigma,K)$ where $\tau = \{F_1, F_{13}, F_{15}, F_{16}\}$, $\tau^c = \{F_{14}, F_2, F_{15}, F_{16}\}$, $\sigma^c = \{F_6, F_5, F_{15}, F_{16}\}$ is defined as $f(F_1) = F_1$, $f(F_2) = F_2$, $f(F_3) = F_3$, $f(F_4) = F_4$, $f(F_5) = F_5$, $f(F_6) = F_6$, $f(F_7) = F_7$, $f(F_8) = F_8$, $f(F_9) = F_9$, $f(F_{10}) = F_{10}$, $f(F_{11}) = F_{11}$, $f(F_{12}) = F_{12}$, $f(F_{13}) = F_{13}$, $f(F_{14}) = F_{14}$, $f(F_{15}) = F_{15}$, $f(F_{16}) = F_{16}$. Clearly f is Soft Slightly A_RScontinuous.

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Example:

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Example:

Let X= $\{x_1, x_2\}$, Y = $\{y_1, y_2\}$ and E= $\{e_1, e_2\}$, K =$\{k_1, k_2\}$ and $f : (X,\tau, E) \rightarrow (Y,\sigma,K)$ where $\tau = \{F_1, F_{13}, F_{15}, F_{16}\}$, $\tau^c = \{F_{14}, F_2, F_{15}, F_{16}\}$, $SA_RSO(X, \tau, E) = \{F_{14}, F_{13}, F_6, F_{12}, F_{11}, F_3, F_8, F_7, F_5, F_4, F_1, F_{15}, F_{16}\}$ and $SA_RSC(X, \tau, E) = \{F_1, F_2, F_3, F_4, F_5, F_6, F_9, F_{10}, F_{11}, F_{12}, F_{14}, F_{15}, F_{16}\}$ and $\sigma = \{F_3, F_{11}, F_{15}, F_{16}\}$, $\sigma^c = \{F_6, F_5, F_{15}, F_{16}\}$is defined as $f(F_1) = F_1$, $f(F_2) = F_2$, $f(F_3) = F_7$, $f(F_4) = F_4$, $f(F_5) = F_5$, $f(F_6) = F_6$, $f(F_7) = F_3$, $f(F_8) = F_8$, $f(F_9) = F_9$, $f(F_{10}) = F_{10}$, $f(F_{11}) = F_{11}$, $f(F_{12}) = F_{12}$, $f(F_{13}) = F_{13}$, $f(F_{14}) = F_{14}$, $f(F_{15}) = F_{15}$, $f(F_{16}) = F_{16}$. Clearly f is Soft Slightly A_RScontinuous. But F_7 is not in SA_RSC of X.

Example:

In thesoft topological space (X, τ, E), (Y, σ,K). X={x_1, x_2} E={e_1,e_2}, Y = {y_1, y_2} , K ={k_1, k_2} and f : (X,τ, E) → (Y, σ,K) , g:(Y,σ,K) → (Z, η,R) where τ = { F_1, F_{13}, F_{15}, F_{16}}, τ^c ={ F_{14}, F_2, F_{15}, F_{16}} then SA_RSO(X, τ, E) ={ F_{14}, F_{13}, F_6 , F_{12} ,F_{11}, F_3, F_8, F_7, F_5, F_4, F_1,F_{15}, F_{16}} and σ = { F_3, F_{11}, F_{15}, F_{16}}, σ^c ={ F_6, F_5, F_{15}, F_{16}} then SA_RSO(Y, σ,K) ={ F_{14}, F_{12} , F_{11}, F_3, F_{10}, F_9, F_8, F_7, F_4, F_7, F_4, F_2, F_1,F_{15}, F_{16}} is defined as $(g \circ f)$ (F_1) = F_1, $(g \circ f)$ (F_2) = F_2, $(g \circ f)$ (F_3) = F_3, $(g \circ f)$ (F_4) = F_4,$(g \circ f)$ (F_5) = F_5, $(g \circ f)$ (F_6) = F_6, $(g \circ f)$(F_7) = F_7, $(g \circ f)$$F_8$) = F_8, $(g \circ f)$ (F_9) = F_9,$(g \circ f)$ (F_{10}) = F_{10},$(g \circ f)$ (F_{11}) = F_{11},$(g \circ f)$(F_{12}) = F_{13},$(g \circ f)$ (F_{13}) = F_{12},$(g \circ f)$ (F_{14}) = F_{14},$(g \circ f)$ (F_{15}) = F_{15},$(g \circ f)$ (F_{16}) = F_{16}. Clearly f and g is Soft slightly A_RS continuous. But ∘ f : (X,τ, E) → (X,τ, E) is not Soft A_RS continuous. Since $(g \circ f)^{-1}$ (F_2) = F_2, $(g \circ f)^{-1}$ (F_9) = F_9, $(g \circ f)^{-1}$ (F_{10}) = F_{10}. F_2, F_9, F_{10}, is not Soft A_RS open in(X,τ, E). Hence $g \circ f$ is not Soft slightly A_RS continuous.

Conclusion:

We introduced Soft Slightly A_RS continuous and studied their properties. By suitable Propositions and examples We established the relations between Soft slightly A_RS continuous and other Soft continuous forms. We hope that these findings paved a new pathway to the researchers in this field .This study not only having the theoretical face but also applied in various scenario of reallife.

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