ON- REGULARITY OF BLOCK TRIANGULAR FUZZY MATRIX

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ABSTACT

Necessary and sufficient conditions are given for the regularity of block triangular matrices ..Equivalent condition for a regular block triangular fuzzy matrix to be expressed as a sum of regular block fuzzy matrices is derived.Further fuzzy relational equations consistency is studied.As an application the existence of group inverse of a block triangular fuzzy matrix is discussed.

KEYWORDS

Fuzzy matrices,regular matrix,semi inverse matrix,block triangular matrix.

INTRODUCTION

This paper aims to assist social scientists to analyze their problems using fuzzy models. The basic and essential fuzzy matrix theory is given. The paper does not promise to give the complete properties of basic fuzzy theory or basic fuzzy matrices. Instead, the authors have only tried to give those essential basically needed to develop, the fuzzy model. The authors do not present elaborate mathematical theories to work with fuzzy matrices. Instead they have given only the needed properties by way of examples. The authors feel that the paper should mainly help social scientists, who are interested in finding out ways to emancipate the society. Everything is kept at simplest level and even difficult definitions, have been omitted. Another main feature of this paper is the description of each fuzzy model using examples from real-word problems. Further. This paper gives lots of reference so that the interested reader can make use ofthem.

PRELIMINARIES

Let F[0, 1] be fuzzy algebra over the support [0, 1] with operations + and - defined as a+b= max{a,b} and a-b= min{a,b}

for all a, b □ [0,1] and then standard order □ . Let F_{mn} be the set of all

m□ n fuzzy matrices over F.

A matrix A□ F_{mn} is set to be regular if there exists X □ F_{mn}

such that AXA =A. In this case X is called a generalized inverse, each element a□F is regular, becauseAx A = A holds under the fuzzy multiplication for all x□ a. Hence F is regular, it is well known that for arbitrary ring R, R is regular. However, the study on regularity of fuzzy matrices. Algorithms for a fuzzy matrix to be regular is given, finite fuzzy relational equation can be expressed in the norm of fuzzy matrix equations as x. A=b, for

some co-efficient matrix A□ F_{mn} and b □ F_{In} .

In the solution of a fuzzy matrix equation whose co-efficient matrix is regular has been discussed. If A is regular with the generalized inverse X then b. X is solution of x.A=b. Further every invertible matrix is regular. Regular fuzzy matrix plays an important role in estimation and inverse problem in fuzzy relation equation and fuzzy optimization problems. In fuzzy retrieval system the degree of relevance of the concept matrix depends on that of its transitive closure which is a regular matrix.

DEFINITION

The matrix $A \in F_{mn}$ is called a regular matrix if there exists a matrix $X \in F_{mn}$ satisfying $AXA=A$. If $AXA=A$, Then X is called generalized (g^{-}) inverse of A and is denotes as A^{-}.

If for the above $X \in F_{mn}$ the equality XAX=X also holds then X is called a semi-inverse of A and is denotes as A^{-}.

LEMMA

For $A, B \in F_{mn}$ the following statements hold

(i) $R(B) \subseteq R(A) \Leftrightarrow$ There exists $X \in F_{mn}$ such that B=XA.

(ii) $C(B) \subseteq C(A) \Leftrightarrow$ There exists $Y \in F_{mn}$ such that B=AY.

REGULAR BLOCK TRIANGULAR FUZZY MATRICES

In this section we derive equivalent conditions for regularity of block triangular fuzzy matrix of the form,

$$M = \begin{bmatrix} A & 0 \\ C & D \end{bmatrix} \text{ With } R(C) \subseteq R(A) \text{ and } C(C) \subseteq C(D) \ldots (1)$$

First, we prove certain lemmas that simplify the proofs of the main results. "I" denote the identity matrix of appropriate size.

LEMMA

For A,B□ F_{mn} if A is regular then

(i) R (B) □ R(A) □ B = B$A^{\square}$ A for each $A^{\square}$ of A.
(ii) C (B) □ `C (A) □ B = A$A^{\square}$ B for each $A^{\square}$ of A.

PROOF:

(i) According to lemma R (B) □ R (A)
□ there exist $X \square F_{mm}$ such that

B =XA. By definition A=A $A^{\square}$ A.

Hence B=XA□ B =X.A$A^{\square}$ A=B $A^{\square}$ A conversely,

i) Suppose B=B $A^{\square}$ A then we have B =XA By taking X=B $A^{\square}$.

Hence R (B) □ R(A) □ B=B$A^{\square}$ A.

Can be proved in the same manner.

LEMMA

Let M= $\square\begin{matrix} A & 0 \\ C & D \end{matrix}\square$ be a lower block triangular matrix. The following statements hold.

(i) If R(C) □ R(A) then $\square_r(M) \square \square_r(A) \square \square_r(D)$

(ii) If C(C) □ C(D) then $\square_c(M) \square \square_c(A) \square \square_c(D)$.

PROOF

From lemma it follows that if R(C) ⊆ R(A) thenthere exist X such that C=XA we express M in the form,

$$M=\begin{bmatrix} I & 0 \\ X & I \end{bmatrix}\begin{bmatrix} A & 0 \\ 0 & D \end{bmatrix}=UL$$

Where, $U=\begin{bmatrix} I & 0 \\ X & I \end{bmatrix}$ and $L=\begin{bmatrix} A & 0 \\ 0 & D \end{bmatrix}$

$$\rho_r(M)=\rho_r(UL)\le\rho_r(L)=\rho_r(A)+\rho_r(D)$$

Since $M=\begin{bmatrix} A & 0 \\ 0 & D \end{bmatrix}$ we get $\rho_r(M)\ge\rho_r(A)+\rho_r(D)$

Hence $\rho_r(M)=\rho_r(A)+\rho_r(D)$

(ii) Can be proved in the same manner.

THEOREM

For any block triangular matrix M of the form M is a rectangular matrix M has a lower block triangular –inverse iff the blocks A and D are regular matrices.

Furthermore

$$\rho(M)=\rho(A)+\rho(D) \quad \text{and } M^{-}=\begin{bmatrix} A^{-} & 0 \\ -D^{-}CA^{-} & D^{-} \end{bmatrix} \text{......... (2)}$$

PROOF:

Since $M^{□}$ is a lower block triangular g-inverse of M of the form (2) and $M^{□}MM^{□}$ is a semi inverse of M equality the corresponds blocks in $M^{□}MM^{□}$ $□ M^{□}$

We get $A^{□} AA^{□} □ A^{□}$ and $D^{□} DD^{□} □ D^{□}$.

Hence $A^{□}$ and $D^{□}$ are inverse of A and D respectively. Then

$M □ □ D^{□}CA^{□} □ A^{□} 0 □$ is a semi inverse of M

$D^{□} □$

$□ □$

It is a clear that if M is regular, then it has a g-inverse and a semi inverse.

For any fuzzy matrix if the Moore Penrose exists. Then by theorem it is unique and coincides with its transpose.

Hence a lower block triangular fuzzy matrix cannot have a lower block triangular Moore Penrose inverse.

Hence A and D are regular matrices.

DEFINITION:4.7

A group inverse $M^{□}$ of a fuzzy matrix M is a semi inverse of M such that M $M^{□}$ = $M^{□}$ M. If $M^{□}$ exists then it is unique.

THEOREM

Let M be of the form M = $\begin{bmatrix} A & 0 \\ C & D \end{bmatrix}$, A and D be square matrices.

Then the group inverse $M^{\#}$ exists if and only if the group inverse $A^{\#}$ and $D^{\#}$ exists and DC=CA.

PROOF:

Since M has the form M = $\begin{bmatrix} A & 0 \\ C & D \end{bmatrix}$, it follows from theorem, and corollary that,

$$M M^{\#} = \begin{bmatrix} AA^{\#} & 0 \\ CA^{\#} & DD^{\#} \end{bmatrix} \text{ and } M^{\#} M = \begin{bmatrix} A^{\#}A & 0 \\ D^{\#}C & D^{\#}D \end{bmatrix}$$

According to the definition of group inverse in $M^{\#}$ exists, then M $M^{\#}$ = $M^{\#}$ M.

That is $AA^{\#} = A^{\#} A$, $D^{\#}D = DD^{\#}$ and $CA^{\#} = D^{\#}C$

It means that $A^{\#}$ and $D^{\#}$ exist and

DC = D ($CA^{\#}$ A) = D ($CA^{\#}$)A = D ($D^{\#}C$ A) = ($DD^{\#}C$) A = CA

The converse follows by retracting the steps.

FUZZY RELATIONAL EQUATIONS

In this section we discuss consistency of fuzzy matrix equation

x.M=b.

Where M = $\begin{bmatrix} A & 0 \\ C & D \end{bmatrix}$ is a lower block triangular fuzzy matrix, and X=[y z] and b=[c d] are partitions of x and b respectively in conformity that of M.

THEOREM

For the matrix M = $\begin{bmatrix} A & 0 \\ C & D \end{bmatrix}$ such that $R(C)$ □ $R(A)$ the blocks A and Dare regular matrices then the following statements are equivalent.

(i) x.M=b is solvable.
(ii) y.A=c, z.D=d are solvable and
(iii) c □ $dD^{□}C$.

PROOF:

(i) □ (ii)

Suppose x.M=b is solvable. Let □ □ [□ □] be a solution.

The sub suit gives □.A □ □ .C □ c and □ .D □ d

Since R(C) □ R (A), by using c=$CA^{□}$ A We get (□ □ □ $CA^{□}$) A =c and□ .D □ d .

Therefore

y.A=c and z.D=d are both solvable with the fact that
y= □ □ □ *CA*□ is a solution of y.A =c and Z= □ is solution of z.D=d

Since D is regular □ □ *dD* is solution of z.D=d.

Now □ *C* □ from □ *A* □ □ *C*=c.
dD□*C*

By fuzzy addition we get $c \square \square\ C \square\ dD^{\square}C$ as required.

Hence (ii) $\square$ (i)

Suppose y.A=c and z.D=d are solvable.

Since both A and D are regular matrices, y= C $A^{\square}$ and z=d $D^{\square}$ are the solutions of the equation y.A=c and z.D = d respectively.

Hence C $A^{\square}$ A=c and $D^{\square}$ D=d.

By fuzzy addition, $c \square dD^{\square}C$ implies $C \square dD^{\square}C \square c$.

$$[c\ A^{\square}\ d\ D^{\square}] \begin{bmatrix} A & 0 \\ C & D \end{bmatrix} = [CAA^{\square} \square dD^{\square}CdD^{\square}D]$$

$$= [C \square dD^{\square}C.d]$$

= [c d] = b

Thus $[cA^{\square}dD^{\square}]$ is a solution of the equation x.M=b

Hence x.M=b is solvable

CONCLUSION

In this see we give some basic matrix theory essential to make the book a self-contained one. However, the book of Paul. Horst on matrix algebra for social scientists would be a boon to social scientists who wish to make use of matrix theory in their analysis.

We give some very basic matrix algebra. This is need for the development of fuzzy matrix theory and the psychological problems.

However, these fuzzy models have been used by applied mathematicians, to study social and psychological problems. These models are very much used by doctors, engineers, scientists, industrialists and statisticians. Here we proceed on to give some basic properties of matrix theory.

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