## ON- REGULARITY OF BLOCK TRIANGULAR FUZZY MATRIX

K.GANAMBIGAI<sup>1</sup>,Dr.S.SANGEETHA<sup>2</sup>,Dr.S.SUBRAMANI<sup>3</sup>,B.APSANA BEGAM<sup>4</sup>
Department of Mathematics
Dhanalakshmi Srinivasan College of
Arts and Science for Women (Autonomous)
Perambalur

#### **ABSTACT**

Necessary and sufficient conditions are given for the regularity of block triangular matrices .. Equivalent condition for a regular block triangular fuzzy matrix to be expressed as a sum of regular block fuzzy matrices is derived. Further fuzzy relational equations consistency is studied. As an application the existence of group inverse of a block triangular fuzzy matrix is discussed.

#### **KEYWORDS**

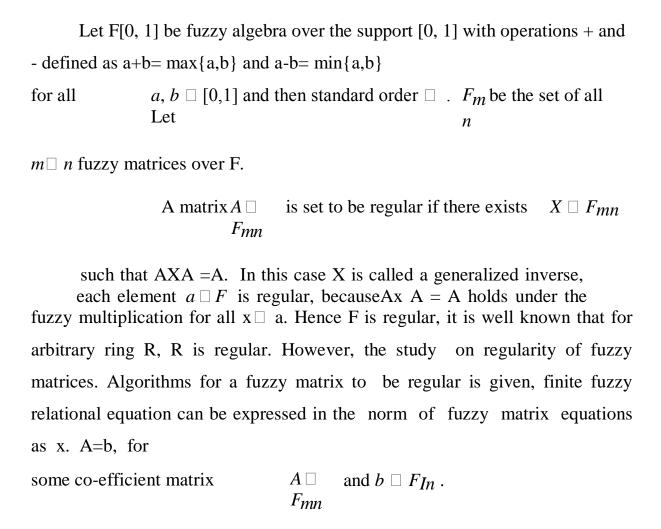
Fuzzy matrices,regular matrix,semi inverse matrix,block triangular matrix.

#### INTRODUCTION

This paper aims to assist social scientists to analyze their problems using fuzzy models. The basic and essential fuzzy matrix theory is given. The paper does not promise to give the complete properties of basic fuzzy theory or basic fuzzy matrices. Instead, the authors have only tried to give those essential basically needed to develop, the fuzzy model. The authors do not present elaborate mathematical theories to work with fuzzy matrices. Instead they have given only the needed properties by way of examples. The authors feel that the paper should mainly help social scientists, who are interested in finding out ways to emancipate the society. Everything is kept at simplest level and even difficult definitions, have been omitted. Another main feature of this paper is

the description of each fuzzy model using examples from real-word problems. Further. This paper gives lots of reference so that the interested reader can make use ofthem.

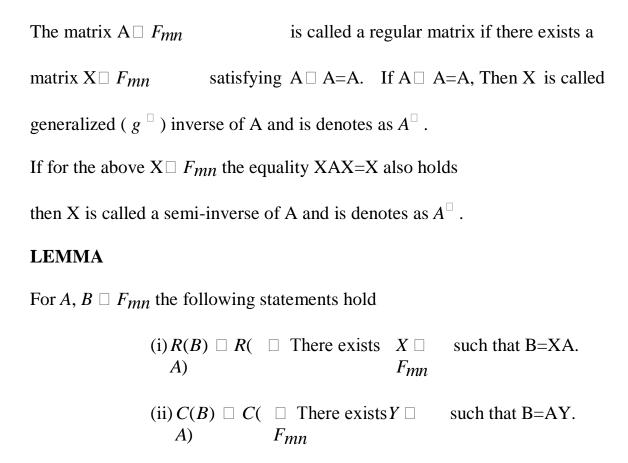
#### **PRELIMINARIES**



In the solution of a fuzzy matrix equation whose co-efficient matrix is regular has been discussed. If A is regular with the generalized inverse X then b. X is solution of x.A=b. Further every invertible matrix is regular. Regular fuzzy matrix plays an important role in estimation and inverse problem in fuzzy relation equation and fuzzy optimization problems. In fuzzy retrieval

system the degree of relevance of the concept matrix depends on that of its transitive closure which is a regular matrix.

#### **DEFINITION**



## REGULAR BLOCK TRIANGULAR FUZZY MATRICES

In this section we derive equivalent conditions for regularity of block triangular fuzzy matrix of the form,

$$M = \begin{array}{ccc} & A & 0 & \square & \text{With } R(C) & \square & R(A) \text{ and } C(C) & \square & C(D)....(1) \\ & & & & & \square & \\ & & & & & \square & \end{array}$$

First, we prove certain lemmas that simplify the proofs of the main results. "I" denote the identity matrix of appropriate size.

# LEMMA

	For $A,B \square$ if A is regular then $F_{mn}$		
	(i) $R(B) \square R(A) \square B = BA^{\square} A \text{ for each } A^{\square} \text{ of } A.$ (ii) $C(B) \square C(A) \square B = AA^{\square} B \text{ for each } A^{\square} \text{ of } A.$		
	PROOF:		
(i)	According to lemma R (B) $\square$ R (A) $\square$ there exist $\square$ such that $\square$		
	B =XA. By definition A=A $A^{\square}$ A.		
	Hence $B=XA \square B=X.AA^{\square}A=BA^{\square}A$ conversely,		
i)	Suppose $B=BA^{\square}$ A then we have $B=XA$ By taking $X=BA^{\square}$ .		
	Hence R (B) $\square$ R(A) $\square$ B=BA $^{\square}$ A.		
	Can be proved in the same manner.		
	LEMMA		
	Let $M = \Box A$ $O \Box$ be a lower block triangular matrix. The $C D \Box$		
	following statements hold.		
(i)	If $R(C) \square R(A)$ then $\square_r(M) \square \square_r(A) \square \square_r(D)$		
(ii)	If $C(C) \square C(D)$ then $\square_c(M) \square \square_c(A) \square \square_c(D)$ .		

## **PROOF**

From lemma it follows that if  $R(C) \square R(A)$  then there exist X such that

C=XA we express M in the form,  $M = \begin{bmatrix} I & 0 & 0 & 0 \\ A & 0 & 0 & 0 \end{bmatrix} = UL$   $X \qquad I = \begin{bmatrix} 0 & 0 & D & 0 \\ A & 0 & D & 0 \end{bmatrix}$ Where,

Where, 
$$U = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$
 and  $U = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ 

$$\Box_{r}(M) \Box_{r}(UL) \Box_{r}(L) \Box_{r}(A) \Box_{r}(D)$$
Since M= 
$$\Box A \qquad 0 \Box \text{ we get } \Box (M_{p}) > \Box (A) \Box \Box (D)$$

Hence 
$$\Box_r(M) \Box \Box_r(A) \Box \Box_r(D)$$

(ii) Can be proved in the same manner.

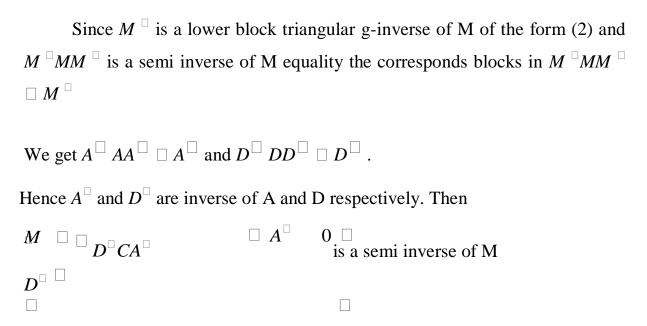
## **THEOREM**

For any block triangular matrix M of the form M is a rectangular matrix M has a lower block triangular –inverse iff the blocks A and D are regular matrices.

Furthermore

$$(M) \square \square (A) \square \square (D) \qquad \qquad \text{and } M = \square A^{\square} \qquad 0 \square \\ D^{\square}CA^{\square} \qquad D^{\square} \square \dots \dots (2)$$

## **PROOF:**



It is a clear that if M is regular, then it has a g-inverse and a semi inverse.

For any fuzzy matrix if the Moore Penrose exists. Then by theorem it is unique and coincides with its transpose.

Hence a lower block triangular fuzzy matrix cannot have a lower block triangular Moore Penrose inverse.

Hence A and D are regular matrices.

## **DEFINITION:4.7**

A group inverse  $M^{\square}$  of a fuzzy matrix M is a semi inverse of M such that M  $M^{\square} = M^{\square}$  M. If  $M^{\square}$  exists then it is unique.

## **THEOREM**

Let M be of the form M = 
$$\Box$$
 A  $0 \Box$ , A and D be square  $C\Box$  D

matrices.

Then the group inverse  $M^{\square}$  exists if and only if the group inverse  $A^{\square}$  and  $D^{\square}$  exists and DC=CA.

## **PROOF:**

Since M has the form 
$$M = \begin{bmatrix} \Box A & 0 & \Box \\ \Box & C & D \end{bmatrix}$$
, it follows from theorem,

and corollary that,

According to the definition of group inverse in  $M^{\square}$  exists, then M  $M^{\square}$  =  $M^{\square}$  M.

That is 
$$AA^{\square} = A^{\square} A$$
,  $D^{\square}D = DD^{\square}$  and  $CA^{\square} \square D^{\square}C$ 

It means that  $A^{\square}$  and  $D^{\square}$  exist and

$$DC = D(CA^{\square} A) = D(CA^{\square})A = D(D^{\square}CA) = (DD^{\square}C)A = CA$$

The converse follows by retracting the steps.

## FUZZY RELATIONAL EQUATIONS

In this section we discuss consistency of fuzzy matrix equation

x.M=b.

Where $M = \begin{bmatrix} A \\ \end{bmatrix}$ is a lower block triangular fuzzy matrix,				
$C$ $D$ $\Box$ and $X=[y\ z]$ and $b=[c\ d]$ are partitions of x and b respectively in conformity that				
of M.				
THEOREM				
For the matrix $M = \begin{bmatrix} \Box A \\ & \Box \\ & \Box C \end{bmatrix}$ such that $R(C) \Box R(A)$ the blocks				
A and Dare regular matrices then the following statements are equivalent.				
(i) x.M=b is solvable.				
(ii) y.A=c, z.D=d are solvable and (iii) $c \Box dD^{\Box}C$ .				
PROOF:				
(i) □ (ii)				
Suppose x.M=b is solvable. Let $\Box$ $\Box$ $\Box$ $\Box$ be a solution.				
The sub suit gives $\Box .A \Box \Box .C \Box c$ and $\Box .D \Box d$				
Since R(C) $\Box$ R (A), by using c= $CA^{\Box}$ A We get ( $\Box$ $\Box$ $CA^{\Box}$ ) $A$ =c and $\Box$ $D$ $\Box$ $d$ .				
Therefore				

	d are both solvable w		
$y = \Box \Box \Box CA^{\Box}$	is a solution of y.A =c and Z= $\square$ is solution of		
z.D=d			
Since D is regular $\Box$ $dD$ is solution of z.D=d.			
	Now $\square C \square$	from $\Box$ $A\Box$ $\Box$ $C=c$ .	
	$dD^{\sqcup}C$		

By fuzzy addition we get  $c \square \square C \square dD^{\square}C$  as required.

Hence (ii)  $\Box$  (i)

Suppose y.A=c and z.D=d are solvable.

Since both A and D are regular matrices,  $y=CA^{\square}$  and  $z=dD^{\square}$  are the solutions of the equation y.A=c and z.D = d respectively.

Hence  $CA^{\square}$  A=c and  $D^{\square}$  D=d.

By fuzzy addition,  $c \square dD^{\square}C$  implies  $C \square dD^{\square}C \square c$ .

$$\begin{bmatrix} c A^{\square} d D^{\square} \end{bmatrix} \stackrel{\square}{=} A \stackrel{\square}{=} \begin{bmatrix} CAA^{\square} \square dD^{\square}CdD^{\square}D \end{bmatrix}$$
$$= \begin{bmatrix} C \square dD^{\square}C.d \end{bmatrix}$$

$$= [c d] = b$$

Thus  $[cA^{\Box}dD^{\Box}]$  is a solution of the equation x.M=b

Hence x.M=b is solvable

#### **CONCLUSION**

In this see we give some basic matrix theory essential to make the book a self-contained one. However, the book of Paul. Horst on matrix algebra for social scientists would be a boon to social scientists who wish to make use of matrix theory in their analysis.

We give some very basic matrix algebra. This is need for the development of fuzzy matrix theory and the psychological problems.

However, these fuzzy models have been used by applied mathematicians, to study social and psychological problems. These models are very much used by doctors, engineers, scientists, industrialists and statisticians. Here we proceed on to give some basic properties of matrix theory.

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