NORMAL FUZZY SUB GROUPS

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ABSTRACT

Given a fuzzy subgroups μ of group G, one define that fuzzy left cosets and the fuzzy right cosets of G relative to μ. We now define hyper fuzzy left cosets and hyper fuzzy right cosets analogously.

INTRODUCTION

In this paper we discuss about fuzzy subgroups μ of group G, one define that fuzzy left cosets and the fuzzy right cosets of G relative to μ. We now define hyper fuzzy left cosets and hyper fuzzy right cosets analogously.

KEY WORDS

Hyper fuzzy, Sub group, Sub set, Co-set

DEFINITION:

Let $\hat{\mu}$ be a hyper fuzzy subgroup of a group G. for any $x \epsilon G$.

Define a mapping $\hat{\mu}_{L(x)}: G \to P^*([0,1])$ by

$$\hat{\mu}_{L(x)}(g) = \hat{\mu}(x^{-1}g) \quad \forall\, g \epsilon G$$

And also define a mapping $\hat{\mu}_{R(x)}: G \to P^*([0,1])$ by

$$\hat{\mu}_{R(x)}(g) = \hat{\mu}(gx^{-1}) \quad \forall\, g \epsilon G$$

Then $\hat{\mu}_{L(x)}$, $\hat{\mu}_{R(x)}$ are respectively called hyper fuzzy left coset and hyper fuzzy left coset and right coset of group G determined by x and $\hat{\mu}$

In crisp concept a subgroup H of a group G for which aH = Ha holds for all $a \epsilon G$. (i.e) left coset equals to corresponding right coset is called normal subgroup of G.

Here we extend this concepts for hyper fuzzy set.

A hyper fuzzy subgroup $\hat{\mu}$ of a group G is called normal if $x, g \epsilon G$.

$$inf\ \hat{\mu}_{L(x)}(g) = inf\ \hat{\mu}_{R(x)}(g)$$

And

$$sup\ \hat{\mu}_{L(x)}(g) = sup\ \hat{\mu}_{R(x)}(g)$$

$$\inf \hat{\mu}(x^{-1}g) = \inf \hat{\mu}(gx^{-1})$$

$$\sup \hat{\mu}(x^{-1}g) = \sup \hat{\mu}(gx^{-1})$$

So we gave formal definition of hyper fuzzy normal subgroup as follows.

DEFINITION:

Let $\hat{\mu}$ be a fuzzy subgroup of a group G. then $\hat{\mu}$ is called a hyper fuzzy normal subgroup of G if

$\inf \hat{\mu}(xy) = \inf \hat{\mu}(yx)$ and $\sup \hat{\mu}(xy) = \sup \hat{\mu}(yx)\ \forall\ x, y \epsilon G$.

PROPOSITION:

The intersection of any two hyper fuzzy normal subgroups of a group G is also a hyper fuzzy normal subgroup of G.

PROOF:

Let $\hat{\mu}$ and $\hat{v}$ be two hyper subgroups of a group G.

$\hat{\mu} \cap \hat{v}$ is hyper fuzzy subgroups of a graph G.

Let $x, y \in G$ then by definition

$$\inf\ (\hat{\mu} \cap \hat{v})(xy) = \min\{\inf \hat{\mu}(xy), \inf \hat{v}(xy)\}$$

$$= \min\{\inf \hat{\mu}\ (xy), \inf \hat{v}\ (yx)$$

$$= \inf(\hat{\mu} \cap \hat{v})(yx)$$

Similarly,

$$sup(\hat{\mu} \cap \hat{v})(xy) = sup(\hat{\mu} \cap \hat{v})(yx)$$

This show that $(\hat{\mu} \cap \hat{v})$ is a hyper fuzzy normal subgroup of G. hence the proposition is proved.

Hence the proposition is proved.

PROPOSITION:

Let $\hat{\mu}$ be hyper fuzzy subgroups of a group G and $a \in G$ then the hyper fuzzy subset $\hat{\mu}: G \to P^*([0,1])$ defined by $\hat{v}(x) = \hat{u}(ax^{-1}a)$. $\forall\, x \in G$ is hyper fuzzy subgroup of G.

PROOF:

Let $x, y \in G$. then for all $a \in G$

$$\inf \hat{v}(xy^{-1}) = \inf(a^{-1}xy^{-1}a) \text{ by definition of } \hat{v}$$

$$= \inf \hat{\mu}\ (a^{-1}xaa^{-1}y^{-1}a)$$

$$= \inf \hat{\mu}\ ((a^{-1}xa)(a^{-1}ya)^{-1})$$

$$\geq \min\{inf\ \hat{\mu}\ ((a^{-1}xa)\, , \inf \hat{\mu}\ (a^{-1}ya)\}$$

Since $\hat{\mu}$ is a hyper fuzzy subgroup.

$$= \min\{\inf \hat{v}(x)\, , \inf \hat{v}(y)\}$$

Again,

$$\sup \hat{v}(xy^{-1}) = \sup \hat{\mu}(a^{-1}xa), \text{ by definition of } \hat{v}$$

$$= \sup \hat{\mu}\,(a^{-1}xaa^{-1}y^{-1}a)$$

$$= \sup \hat{\mu}((a^{-1}xa)(a^{-1}ya)^{-1})$$

$$\geq \min\{sup\,\hat{\mu}\,(a^{-1}xa), \sup \hat{\mu}\,(a^{-1}ya)\}$$

Since $\hat{\mu}$ is a hyper fuzzy subgroups.

$$= \min\{\sup \hat{v}(x), \sup \hat{v}(y)\}$$

Hence $\hat{\mu}$ is a hyper fuzzy subgroups of G.

DEFINITION:

Let $\hat{\mu}$ and $\hat{v}$ be two hyper fuzzy subgroups of a group G. we say that $\hat{v}$ is conjugate to $\hat{\mu}$ if for some $a \in G$. we have that

$$\inf \hat{v}(x) = \inf \hat{\mu}(a^{-1}xa) \quad \forall\, x \in G$$

$$\sup \hat{v}(x) = \sup \hat{\mu}(a^{-1}xa) \quad \forall\, x \in G$$

PROPOSITION:

For any hyper fuzzy subset $\hat{\mu}$ of a group G and for all x, y $\epsilon\, G$ and for all $x, y \in G$ following are equivalent.

(i) $\inf \hat{\mu}(xyx^{-1}) = \inf \hat{\mu}(y)$ and
$\sup \hat{\mu}(xyx^{-1}) = \sup \hat{\mu}(y)$

(ii) $\inf \hat{\mu}(xy) = \inf \hat{\mu}(yx)$ and
$\sup \hat{\mu}(xy) = \sup \hat{\mu}(yx)$

(iii) $\inf\ \hat{\mu}_{L(x)}(y) = \inf \hat{\mu}_{R(x)}(y)$ and
$\sup\ \hat{\mu}_{L(x)}(y) = \sup \hat{\mu}_{R(x)}(y)$

PROOF:

Let $x, y \in G$ and be hyper fuzzy subgroups of a group G.

$(i) \Longrightarrow (ii)$

$$\inf \hat{\mu}(y) = \inf \hat{\mu}(x^{-1}xyx)$$

$$= \inf \hat{\mu}(xy) \quad \text{using (i)}$$

And $$\sup \hat{\mu}(yx) = \sup \hat{\mu}(x^{-1}xyx)$$

$$= \sup \hat{\mu}(xy)$$

$(ii) \Longrightarrow (iii)$

$$\inf \hat{\mu}_{L(x)}(y) = \inf \hat{\mu}(x^{-1}y)$$

$$= \inf \hat{\mu}(yx^{-1}) \text{ using (ii)}$$

$$= \inf \hat{\mu}_{R(x)}(y)$$

And $$\sup \hat{\mu}_{L(x)}(y) = \sup \hat{\mu}(x^{-1}y)$$

$$= \sup \hat{\mu}(yx^{-1}) \quad \text{using (ii)}$$

$$= \sup \hat{\mu}_{R(x)}(y)$$

$(iii) \Longrightarrow (i)$

$$\inf \hat{\mu}(xyx^{-1}) = \hat{\mu}_{R(x)}(xy)$$

$$= \inf \hat{\mu}_{L(x)}(xy) \quad \text{using (iii)}$$

$$= \inf \hat{\mu}(x^{-1}xy)$$

$$= \inf \hat{\mu}(y)$$

And $$\sup \hat{\mu}(xyx^{-1}) = \sup \hat{\mu}_{R(x)}(xy)$$

$$= \sup \hat{\mu}_{L(x)}(xy)$$

$$= \sup \hat{\mu}(x^{-1}xy)$$

$$= \sup \hat{\mu}(y)$$

Hence the proposition is proved.

A hyper fuzzy subgroup $\hat{\mu}$ of a group G is called conjugate hyper fuzzy subgroup if for all, $x \epsilon G$ we have that

$$\inf \hat{\mu}\,(x) = \inf \hat{\mu}\,(a^{-1}xa) \text{ and } \sup \hat{\mu}\,(x) = \sup \hat{\mu}\,(a^{-1}xa)$$

PROPOSITION:

A hyper fuzzy subgroup $\hat{\mu}$ of a group G is normal iff $\hat{\mu}$ is self conjugate hyper fuzzy subgroup.

PROOF:

Let $\hat{\mu}$ be a hyper fuzzy normal subgroup of group G. then

$$inf\,\hat{\mu}(xy) = inf\,\hat{\mu}(yx) \text{ and}$$

$$sup\ \hat{\mu}(xy) = sup\hat{\mu}(yx) \quad \forall\ x, y\ \epsilon\ G$$

We have

$$inf\,\hat{\mu}(xyx^{-1}) = inf\,\hat{\mu}(y) \text{ and}$$

$$sup\ \hat{\mu}(xyx^{-1}) = sup\ \hat{\mu}(y)\ \forall\ x, y\ \epsilon\ G$$

So $\hat{\mu}$ is a self conjugate hyper fuzzy subgroup.

Conversely,

Let $\hat{\mu}$ is a self conjugate hyper fuzzy subgroup.

Thus $$inf\,\hat{\mu}(xyx^{-1}) = inf\,\hat{\mu}(y)$$

And $$sup\ \hat{\mu}(xyx^{-1}) = sup\ \hat{\mu}(y)\ \forall\ x, y\ \epsilon\ G$$

We have

$$inf\ \hat{\mu}(xy) = inf\ \hat{\mu}(yx) \text{ and}$$

$$sup\ \hat{\mu}(xy) = sup\ \hat{\mu}(yx)\ \forall\ x, y\ \epsilon\ G$$

So $\hat{\mu}$ is a self conjugate hyper fuzzy normal subgroup

This completes the proof.

DEFINITION:

Let $\hat{\mu}$ hyper fuzzy subgroup of a group G. then normalizer of $\hat{\mu}$ is defined by

$$N(\hat{\mu}) = \{a \in G : \forall \ x \in G, \inf \hat{\mu}(a^{-1}xa) = \inf \hat{\mu}(x),$$

$$sup \ \hat{\mu}(a^{-1}xa) = sup \ \hat{\mu}(x)\}$$

PROPOSITION:

Let $\hat{\mu}$ hyper fuzzy subgroup of a group G. then

(i) $N(\hat{\mu})$ is a subgroup of G

(ii) $\hat{V} : N(\hat{\mu}) \to P^*([0,1])$ is defined by

$$\hat{V}(x) = \hat{\mu}(x) \ \ \forall \ x \in N(\hat{\mu})$$

Then $\hat{V}$ is a hyper fuzzy normal subgroup of $N(\hat{\mu})$

PROOF:

Let $x, y \in N(\hat{\mu})$. Then for all $g \in G$

$$inf \ \hat{\mu}\big(xy^{-1}g(xy)\big) = inf \hat{\mu}(y^{-1}x^{-1}gxy)$$

$$= inf \hat{\mu}(x^{-1}gx)$$

Since $y \in N(\hat{\mu})$, $x^{-1}gx \in G$

$$= inf \hat{\mu}(g) \text{ since } x \in N(\hat{\mu})$$

Similarly,

$$sup \ \hat{\mu}\big(xy^{-1}g(xy)\big) = sup \hat{\mu}(y^{-1}x^{-1}gxy)$$

$$= sup \hat{\mu}(x^{-1}gx)$$

Since $y \in N(\hat{\mu})$, $x^{-1}gx \in G$

$$= sup\hat{\mu}(g), \text{ since } x \epsilon N(\hat{\mu})$$

So $xy \epsilon N(\hat{\mu})$

Again $\quad g \epsilon G, x \epsilon N(\hat{\mu}) = xgx^{-1} \epsilon G$

Then for all $g \epsilon G$

$$inf\hat{\mu}(xgx^{-1}) = inf\hat{\mu}(x^{-1}(xgx^{-1})x)$$

Since $x \epsilon N(\hat{\mu}),\ xgx^{-1} \epsilon G$

$$= inf\ \hat{\mu}(x^{-1}xgx^{-1}x)$$

$$= \inf \hat{\mu}(g)$$

Similarly , then for all $g \epsilon G$

$$\sup \hat{\mu}(xgx^{-1}) = \sup \hat{\mu}(g)$$

So, $x^{-1} \epsilon N(\hat{\mu})$

Hence $N(\hat{\mu})$ is a subgroup of G.

(ii) since $\hat{\mu}$ is a hyper fuzzy subgroup of G and we prove that is a subgroup in G.

Then $\hat{\mu}$ is a hyper fuzzy group of $N(\hat{\mu})$

Hence $(\hat{v})$ is a hyper fuzzy group of $N(\hat{\mu})$

Now we have to prove $\hat{v}$ is a normal.

Since $N(\hat{\mu})$ is a subgroup of G $x, y \epsilon N(\hat{\mu})$

$$x^{-1}yx \epsilon N(\hat{\mu})$$

Now by definition of $\hat{v}$ we have for all $x, y \epsilon N(\hat{\mu})$

$$\inf \hat{v}(x^{-1}yx) = \inf \hat{\mu}(x^{-1}yx)$$

Since $x^{-1}yx \epsilon N(\hat{\mu})$

$$= \inf \hat{\mu}(y), \text{ since } x \epsilon N(\hat{\mu})$$

$$= \inf \hat{v}(y), \text{ since } y \epsilon N(\hat{\mu})$$

Similarly,

$$\sup \hat{v}(x^{-1}yx) = \sup \hat{\mu}(x^{-1}yx)$$

Since $x^{-1}yx \epsilon N(\hat{\mu})$

$$= \sup \hat{\mu}(y),\ \text{ since } x \epsilon N(\hat{\mu})$$

$$= \sup \hat{v}(y),\ \text{ since y } \epsilon N(\hat{\mu})$$

4Hence $\hat{v}$ is a self conjugate hyper fuzzy subgroup of N($\hat{\mu}$).

Hence $\hat{v}$ is a hyper fuzzy normal subgroup of N($\hat{\mu}$)

This completes the proof.

CONCLUSION:

The concept of fuzzy set is very simple and easy to understand. In a short span of time. We have done only a little drops in this field. In this paper we have done the basic definitions of fuzzy concepts on normal and hyper subgroups.

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