KNOT THEORY-SOME BASIC CONCEPTS

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ABSTRACT

A knot made out of a string are often deformed in space without cutting open the closed knot.The deformed knot is taken into account to be an equivalent because the original knot.

INTRODUCTION

In topology, knot theory is that the study of mathematical knots. In the late 1970$'s$ William Thurston introduced non-Euclidean geometry into the study of knots with the hyperbolization theorem.
Early modern, knots were studied from a mathematical viewpoint by Carl Friedrich Gauss, Who in 1833 developed the Gauss linking integral for computing the linking number of two knots..He formulated what are now referred to as the Tait conjectures on alternating knots. (The conjectures were proved in the 1990s.

KEY WORDS

Vertices, Edges, Seifert Circles, Knot.

Theorem

The uniqueness and existence of a decomposition of knots.

1) Any knot can be decomposed into a finite numberofknots.
2) This decomposition excluding the order,is unique.That is if a knot is decomposedintwowaysas$k_1 k_2 k_3,\ldots k_m$and$k'_1 k'_2 k'_3\ldots k'_n$,thenn=m.Also renumbering the subscript suitablyof

$k_1, k_2, k_3, \ldots k_m, k_1 \approx k_1', k_2 \approx k_2', \ldots\ldots, k_m \approx k'_n$

Preposition:2.1.1

$K_1 \# K_2$ is equivalent to $k_2 \neq k_1$, with orientation that is commutative law holds for sum of two knots.

Also associative law holds that is

$$\boldsymbol{k_1 \neq (k_2 \neq k_3) \approx (k_1 \neq k_2) \neq k_3}$$

Let A denote the set of all oriented knots and let sum of knots be defined on A because the connected sum. Then A may be a semi-group but not a gaggle under this operation.A contains the unit element,the trivial knot,O but it doesn't contain inverse elements.For example, for the oriented trefoil knot K there does not exist a knot K' such that $K \neq K' = 0$

Theorem

Suppose K_1 and K_2 are two arbitrary knots,then

$$(k_1 \neq k_2) = (k_1) + b(k_2) - 1$$

Proof:

If a knot $k1$ equivalent to a knot k_2 then $(k_1) = (k_2)$

So the bridge number of a knot K, b(K) is an invariant forK.

Theorem

If c is the number of crossings and s is the number of seifert circles then $\aleph = s - c$ and the genus of the surface is $g = (c - s + 1)/2$.

Proof:

Let the seifert surface have no vertices at each crossings, one an each strand. Join these vertices with edges. This gives c extra edges and two vertices.

Apart from the edges there are two other edges from each vertex which gives 4c edges.

But then, here each edge will be counted twice and so the number of other edges is only 2c.

Thus there are $(c + 2c = 3c)$edges,2c vertices and s faces.

Therefore

$$\aleph = V - E + F = 2c - 3c + s$$

$$= s - c$$

$$g = \frac{1}{2}(1 - \aleph) = \frac{1}{2}(c - s + 1)$$

Thus the seifert algorithm can be used to find the surfaces bounding the knots and the Euler characteristics can be used to identify the type of surfaces.

Reidemeister moves:

A Reidemeister move is one among the 3 ways to vary a projection of the knot which will change the relation between the crossings.

1) FirstReidemeistermove:

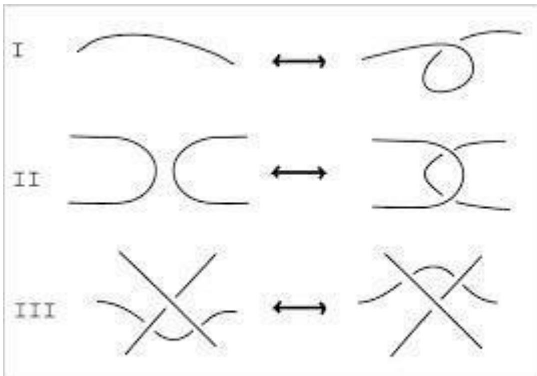
A twist are often put in or taken call at a knot.

2) SecondReidemeistermove:

Two crossings can be added or removed.

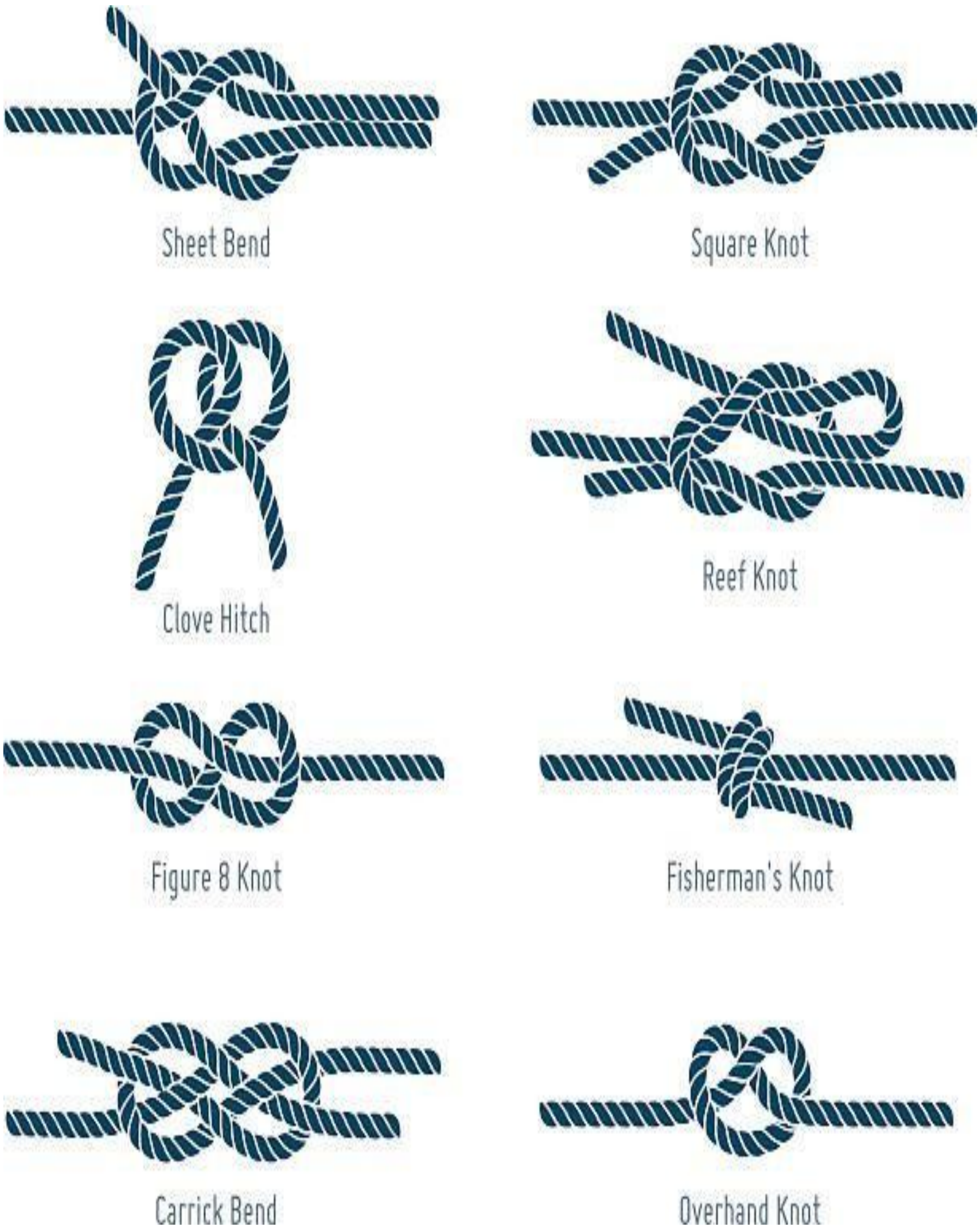
3) Third Reidemeistermove:

A stand of the knot can be slid from one side of the crossing to the other side.



The german mathematician kurt Reidemeister proved that if there are two projections of an equivalent knot,then one are often obtained from the opposite bya series of Reidemeister moves and planarisotopies

List of basic knots:



Theorem

Given an arbitrary oriented knot (or link) K, there exists in R^3 an orientable, Connected surface that has as its boundary K.

Proof:

Seifert gave an algorithm to construct such a surface.

i) To construct an embedded surface in space for a specific knot K.

ii) Starting with a projection of the knot. Choose an orientationonK.

iii) At each crossing of the projection, 2 strands come in and 2 strands go out. Eliminate the crossing by connecting each of the strands coming into the crossing to the adjacent strand leaving the crossing. The resultant strand will no longercross.

iv) The result will be a set of circles in a plane. These circles are called ***Seifert circles***. Fill in the circles to get discs in the plane. The discs are choosen at different heights rather than in the same plane so that they do not intersect eachother.

v) Finally, connect the discs to one another at the crossings of theknot bytwistedbands.

vi) The result is a surface with one boundary component such that theboundary component istheknot.

vii) The surface is also orientable. Let the Seifert circle have orientation of the knot. For each disc that features a clockwise orientation on its bounding Seifert Circle paint its upward pointing face white and its downward pointing faceblack.

viii)For each disc that has counterclockwise orientation on its bounding Seifert Circle, paint its upward pointing face black and its downwardpointing facewhite.

At each crossing, the two discs are connected by a band with a half twist.

If two discs are adjacent then they are of opposite orientation on their boundaries.

If one disc is on top of the other then they are of the same orientation on heir boundaries. Due to the half twist the entire surface can be painted black and white so that no black paint touches any white paint.

Thus the surface is orientable.

Theorem

Applying seifert’s algorithm to an alternating projection of an alternating knot or link does yield a seifert surface of minimal genus.

Proof:

Given by **David Gabai,**

The genus of an unknot is 0.

Thus it bounds a disc which can be capped off to form a sphere.

Note:

1) A trivial knot cannot be the composition of two nontrivial knots. For, suppose that O isanunknot,

then g(O) = 0 and g(K)>0 if K≠O Suppose that O= J#K where J, K≠ 0,

2) The knots of genus 1 are prime.For, let K be a knot suchthat g(K)=1.

If K is not prime,

then K is the composition of two knots, say,K_1 and K_2, ie) $k \approx k_1 + k_2$

Then 1= $g_k = (k_1) + (k_2)$

Therefore either

$(k_1) = 1$and $(k_2) = 0$ or

$(k_1) = 0$ and $(k_2) = 1$.

In either case one of them is an unknot.

Hence k is prime.The Seifert's Algorithm can be used to find a minimal genus Seifert Surface for an alternating knot.

But there are other types of knots too.

An Israeli mathematician, Yoan Moriah proved that there are knots for which minimal genus Seifert Surface cannot be obtained by applying Seifert Algorithm to any projection of the knot.

CONCLUSION

I have made a survey of what has led to the study of knots in mathematics, the relevance of the study, the fundamental concepts which are dealt with in the theory, the slow development of the subject and its application in the inter disciplinary field.

This is a brief report of a small area of the vast field of Knot Theory. The knots can be studied classifying them as torus knots, satellite knots and hyperbolic knots. Every knot or link is proved to be a closed braid.

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