HYPER FUZZY COSETS

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ABSTRACT

We define hyper fuzzy set as a generalization of fuzzy sets and interval valued fuzzy sets. Then we introduce the notion of hyper fuzzy subgroups and study some of its important properties.

INTRODUCTION:

Now we define hyper fuzzy set as a generalization of fuzzy sets and interval valued fuzzy sets. Then we introduce the notion of hyper fuzzy subgroups and study some of its important properties.

KEYWORDS:

Fuzzy set, Fuzzy Subgroups, subgroupoid, Hyper Fuzzy Subgroup.

DEFINITION:

Let X be a set. Then mapping $\hat{\mu} : X \to P^*([0,1])$ is called a hyper fuzzy subset of X where $P^*([0,1])$ denotes the set of all non empty subset of [0,1].

DEFINITION:

Let X be a non empty set and $\hat{\mu}, \hat{\nu}$ be two hyper fuzzy subset of X. then intersection of $\hat{\mu}$ and $\hat{\nu}$ is denote $\hat{\mu} \cap \hat{\nu}$ and defined by

(i) $(\hat{\mu} \cap \hat{\nu})(X) = \{\min\{p, q\}: p \in \hat{\mu}(X), q \in \hat{\nu}(X)\}$ for all x $\in X$

Then union $\hat{\mu}$ and $\hat{\nu}$ is denoted by $\hat{\mu} \cup \hat{\nu}$ and defined by

(ii) $(\hat{\mu} \cup \hat{\nu})(X) = \{\max\{p, q\}: p \in \hat{\mu}(X), q \in \hat{\nu}(X)\}$for all x $\in X$

DEFINITION:

Let X be a groupoid i.e) a set which is closed under a binary relation denoted multiplicatively.

A mapping $\hat{\mu} : X \to P^*([0,1])$ is called a hyper fuzzy subgroupoid if $\forall\ x, y \in X$ following conditions hold.

(iii) $\inf \hat{\mu}(xy) \geq \min\{\inf \hat{\mu}(x), \inf \hat{\mu}(y)\}$
(iv) $\sup \hat{\mu}(xy) \geq \min\{\sup \hat{\mu}(x), \sup \hat{\mu}(y)\}$

DEFINITION:

Let G be a group. A mapping $\hat{\mu} : X \to P^*([0,1])$ is called a hyper fuzzy subgroupoid if $\forall\ x, y \in X$ following conditions hold.

(v) $\inf \hat{\mu}(xy) \geq \min\{\inf \hat{\mu}(x), \inf \hat{\mu}(y)\}$
(vi) $\sup \hat{\mu}(xy) \geq \min\{\sup \hat{\mu}(x), \sup \hat{\mu}(y)\}$
(vii) $\inf \hat{\mu}(x^{-1}) \geq \inf \hat{\mu}\ (x)$
(viii) $\sup \hat{\mu}(x^{-1}) \geq \sup \hat{\mu}\ (x)$

PROPOSITION:

If $\hat{\mu}$ is a hyper fuzzy subgroup of G having the identity e, then for all x ϵ X.

(i) $\inf \hat{\mu}(x^{-1}) \geq \inf \hat{\mu}\ (x)$ and $\sup \hat{\mu}(x^{-1}) \geq \sup \hat{\mu}\ (x)$
(ii) $\inf \hat{\mu}(e) \geq \inf \hat{\mu}\ (x)$ and $\sup \hat{\mu}(e) \geq \sup \hat{\mu}\ (x)$

PROOF:

(i) As $\hat{\mu}$ is a hyper fuzzy subgroups of group G then $\inf \hat{\mu}(x^{-1}) \geq \inf \hat{\mu}\ (x)$ $\forall\ x \in G$

again,

$$\inf \hat{\mu}(x) = \inf \hat{\mu}(x^{-1}) \geq \inf(x^{-1})$$

so,

$$\inf \hat{\mu}(x^{-1}) = \inf \hat{\mu}(x)$$

similarly,

we can prove that

$$\sup \hat{\mu}(x^{-1}) = \sup \hat{\mu}(x)$$

(ii) $$\inf \hat{\mu}(e) = \inf \hat{\mu}(xx^{-1})$$

$$\geq \min\{\inf \hat{\mu}(x), \inf \hat{\mu}(x^{-1})\}$$

$$= \inf \hat{\mu}(x)$$

And $$\sup \hat{\mu}(e) = \sup \hat{\mu}(xx^{-1})$$

$$\geq \min\{\sup \hat{\mu}(x), \sup \hat{\mu}(x^{-1})\}$$

$$= \sup \hat{\mu}(x)$$

Hence the proposition is proved.

PROPOSITION:

A hyper fuzzy subset $\hat{\mu}$ of group G is a hyper fuzzy subgroup iff for all x,y ϵ G following are hold.

(i) $\inf \hat{\mu}(xy^{-1}) \geq \min\{\inf \hat{\mu}(x), \inf \hat{\mu}(y)\}$
(ii) $\sup \hat{\mu}(xy^{-1}) \geq \min\{\sup \hat{\mu}(x), \sup \hat{\mu}(y)\}$

PROOF:

At first let $\hat{\mu}$ be a hyper fuzzy subgroup of G and x,y ϵ G. then

$$\inf \hat{\mu}(xy^{-1}) \geq \min\{\inf \hat{\mu}(x), \inf \hat{\mu}(y^{-1})\}$$

$$= \min\{\inf \hat{\mu}(x), \inf \hat{\mu}(y)\} \text{ and}$$

$$\sup \hat{\mu}(xy^{-1}) \geq \min\{\sup \hat{\mu}(x), \sup \hat{\mu}(y^{-1})\}$$

$$= \min\{\sup \hat{\mu}(x), \sup \hat{\mu}(y)\}$$

Conversely,

Let be a hyper fuzzy subset of G and given condition hold. Then for all x ϵ G we have

$$\inf \hat{\mu}(e) = \inf \hat{\mu}(xx^{-1})$$

$$\geq \min\{\inf \hat{\mu}(x), \inf \hat{\mu}(x)$$

$$= \inf \hat{\mu}(x) - --\rightarrow (1)$$

$$\sup \hat{\mu}(e) = \sup \hat{\mu}(xx^{-1})$$

$$\geq \min\{\sup \hat{\mu}(x), \sup \hat{\mu}(x)$$

$$= \sup \hat{\mu}(x) - --\rightarrow (2)$$

So,

$$\inf \hat{\mu}(x^{-1}) = \inf \hat{\mu}(ex^{-1})$$

$$\geq \min\{\inf \hat{\mu}(e), \inf \hat{\mu}(x)$$

$$= \inf \hat{\mu}(x) - --\rightarrow by\ (1)$$

By

$$\sup \hat{\mu}(x^{-1}) = \sup \hat{\mu}(ex^{-1})$$

$$\geq \min\{\sup \hat{\mu}(x), \sup \hat{\mu}(x)$$
$$= \sup \hat{\mu}(x) - --\rightarrow by\ (2)$$

Again,

$$\inf \hat{\mu}\,(xy) \geq \min\{\inf \hat{\mu}(x), \inf \hat{\mu}(y^{-1})\}$$

Using given condition

$$\geq \min\{\inf \hat{\mu}(x), \inf \hat{\mu}(y)\}$$

And

$$\sup \hat{\mu}\,(xy) \geq \min\{\sup \hat{\mu}(x), \sup \hat{\mu}(y^{-1})\}$$

using given condition

$$\geq \min\{\sup \hat{\mu}(x), \sup \hat{\mu}(y)\} \text{ by above}$$

Hence $\hat{\mu}$ hyper fuzzy subgroup of G.

Hence the proposition is proved.

PROPOSITION:

Intersection of any two hyper fuzzy subgroups of a group G is called also a hyper fuzzy subgroup of G.

PROOF:

Let $\hat{\mu}$ and $\hat{v}$ be two hyper fuzzy subgroups of a group G. $x, y \epsilon G$.

Then

$$\inf(\hat{\mu} \cap \hat{v})(xy^{-1}) = \min\{\inf \hat{\mu}(xy^{-1}), \inf \hat{v}(xy^{-1})$$

$$= \{\min \{\min\{\inf \hat{\mu}(x), \inf \hat{\mu}(y) \,,\, min \,\{\inf \hat{v}(x), \inf \hat{v}(y)\}\}$$

$$= \{\min \{\min\{\inf \hat{\mu}(x), \inf \hat{v}(y) \,,\, min \,\{\inf \hat{\mu}(x), \inf \hat{v}(y)\}\}$$

$$= \min\{\inf(\hat{\mu} \cap \hat{v})(x) \,, \inf(\hat{\mu} \cap \hat{v})(y) \ldots \ldots (3)$$

Again

$$\sup(\hat{\mu} \cap \hat{v})(xy^{-1}) = \min\{\sup \hat{\mu}(xy^{-1}), \sup \hat{v}(xy^{-1})\}$$

$$= \{\min \{\min\{\sup \hat{\mu}(x), \sup \hat{\mu}(y) \,,\, min \,\{\sup \hat{v}(x), \sup \hat{v}(y)\}\}$$

$$= \{\min \{\min\{\sup \hat{\mu}(x), \sup \hat{v}(y) \,,\, min \,\{\sup \hat{\mu}(x), \sup \hat{v}(y)\}\}$$

$$= \min\{\sup(\hat{\mu} \cap \hat{v})(x) \,, \sup(\hat{\mu} \cap \hat{v})(y) \ldots \ldots (4)$$

Hence (3) and (4) using we say that $\hat{\mu} \cap \hat{v}$ is a hyper fuzzy subgroup of G. Hence proposition is proved.

CONCLUSION:

The above study has helped me to derive some equivalent conditions for each fuzzy subgroup. It has helped me to acquire some that basic knowledge of fuzzy. The fundamental concept of fuzzy sets play a prominent role in mathematics with wide application in many other branches such as theoretical physics, computer science, control engineering information science, coding theory , graph theory, real analysis measure theory etc,…

REFFERENCE:

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