Fourier Analysis on Finite Abelian Groups

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ABSTRACT

This article reveals the basic techniques of fourier analysis on a finite abelian group .It may be used as to derive theorems in graph theory.The definition of fourier transform depend on the choice of Isomorphism but it is independent of this choice upto automorphism of R. Fourier transformation has been a rich source for applications in combinatories.

INTRODUCTION:

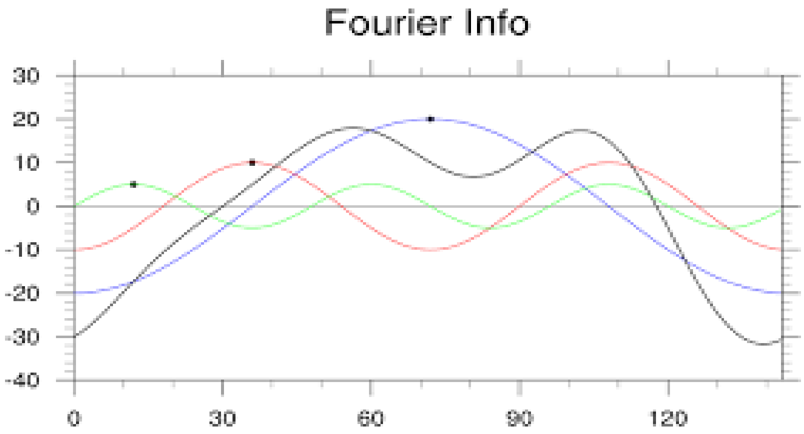
In this chapter we introduce an algebraic structure on L(G) coming from the convolution product. The Fourier transform then permits us to analyse this structure more clearly in terms of known rings. In particular, we prove Wedderburn's theorem for group algebras over the complex numbers. Due to its applications in signal and image processing, Fourier analysis is one of the most important aspects of mathematics. There are entire books dedicated to Fourier analysis on finite groups. Unfortunately, we merely scratch the surface of this rich theory in this text. In particular, the only application that we give is to computing the eigenvalues of the adjacency matrix of a Cayley graph of an abelian group.

KEYWORDS

Abelian Group,homomorphism,automorphism,cyclicgroup.

What is a Fourier Analysis

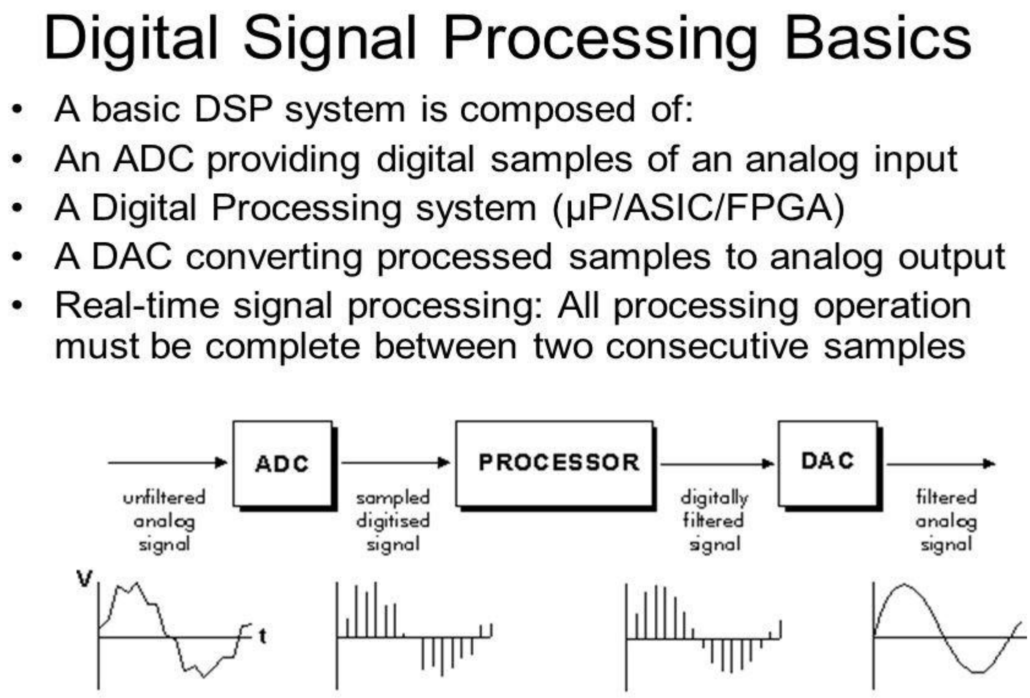
Fourier Analysis is a method of defining Periodic waveforms in terms of trigonometric function.

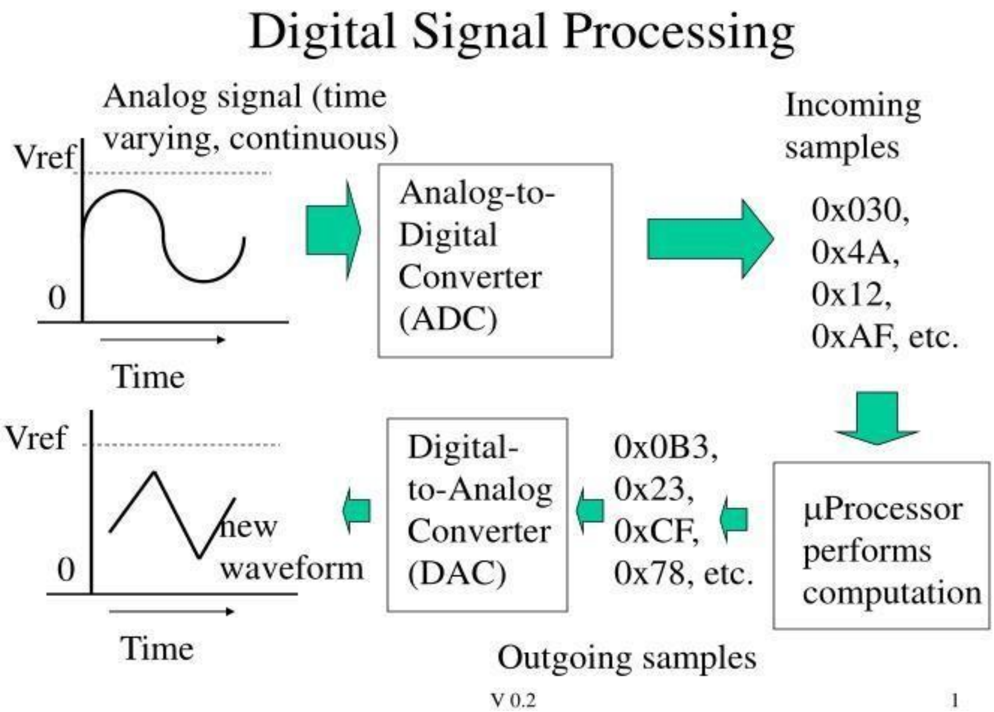


APPLICATION

Signals

When we observe physical signals, changes in air pressure, electromagnetic field, etc., in and log form using some recording device, the recorded signal is only an approximation of the original due to the errors inherent in any sensors.
Likewise, even if we assume that the original physical signal satisfied the requirements of the sampling theorem for exact reproduction, our sampling devices have their inherent errors and, thus, only an approximation of the digital equivalent of the first physical signal are often captured. However, as long because the errors are smaller than the accuracy required for extracting the relevant information the system is ok for practical purposes, and a key element in engineering practice is to balance the cost and performance of the overall system.





Signal processing

When processing signals, like audio, radio waves, light waves, seismic waves, and even images, Fourier analysis can isolate narrowband components of a compound waveform, concentrating them for easier detection or removal. A large family of signal processing techniques contains Fourier transforming a sign , manipulating the Fourier-transformed data during a simple way, and reversing the transformation.

Equalization of audio recordings with a series of bandpass filters;

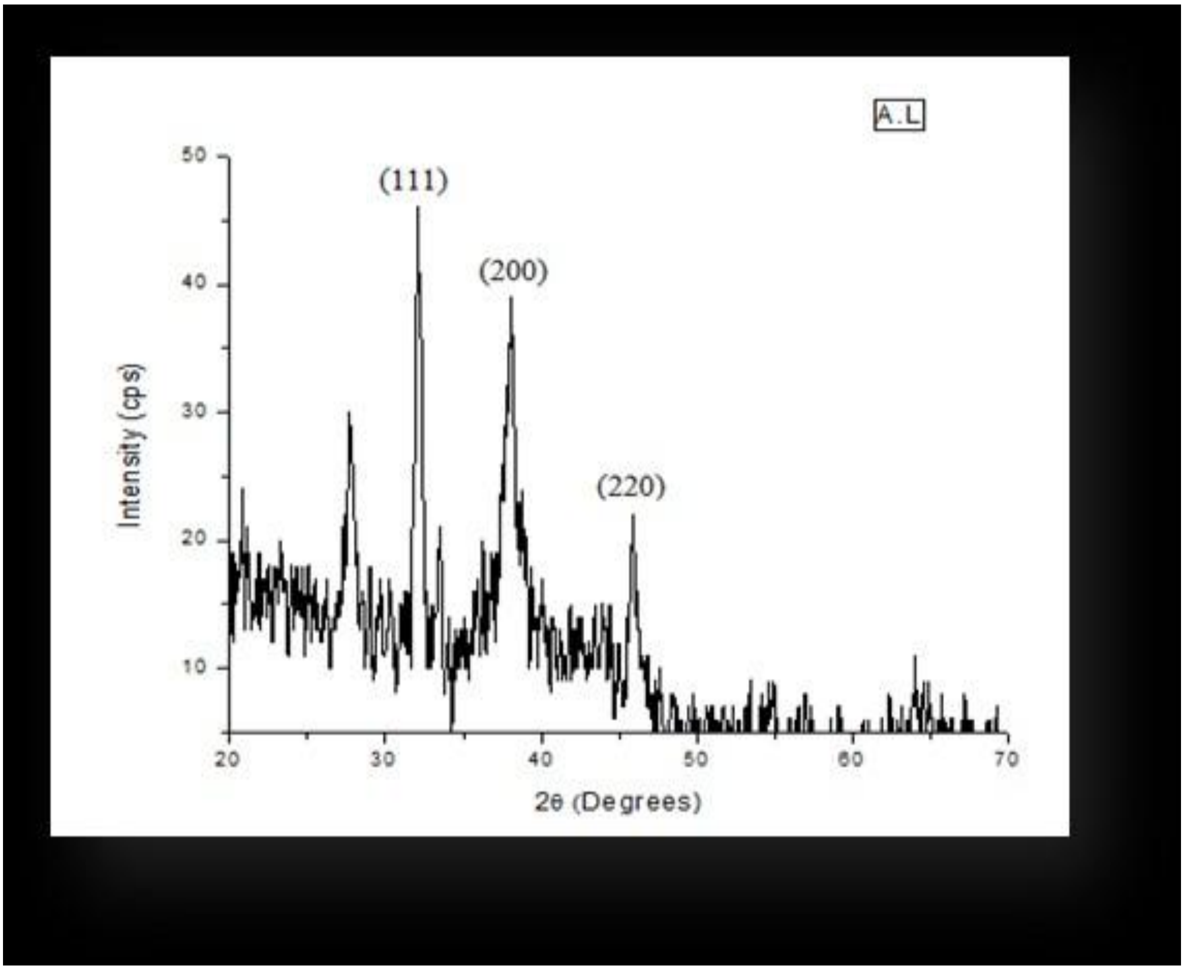
Digital radio reception without an excellent heterodyne circuit, as during a modern telephone or radio scanner;

Image processing to remove periodic or anisotropic artifacts such as jaggies from interlaced video, strip artifacts from stripaerial photography, or wave patterns from radio frequency interference in a digital camera;

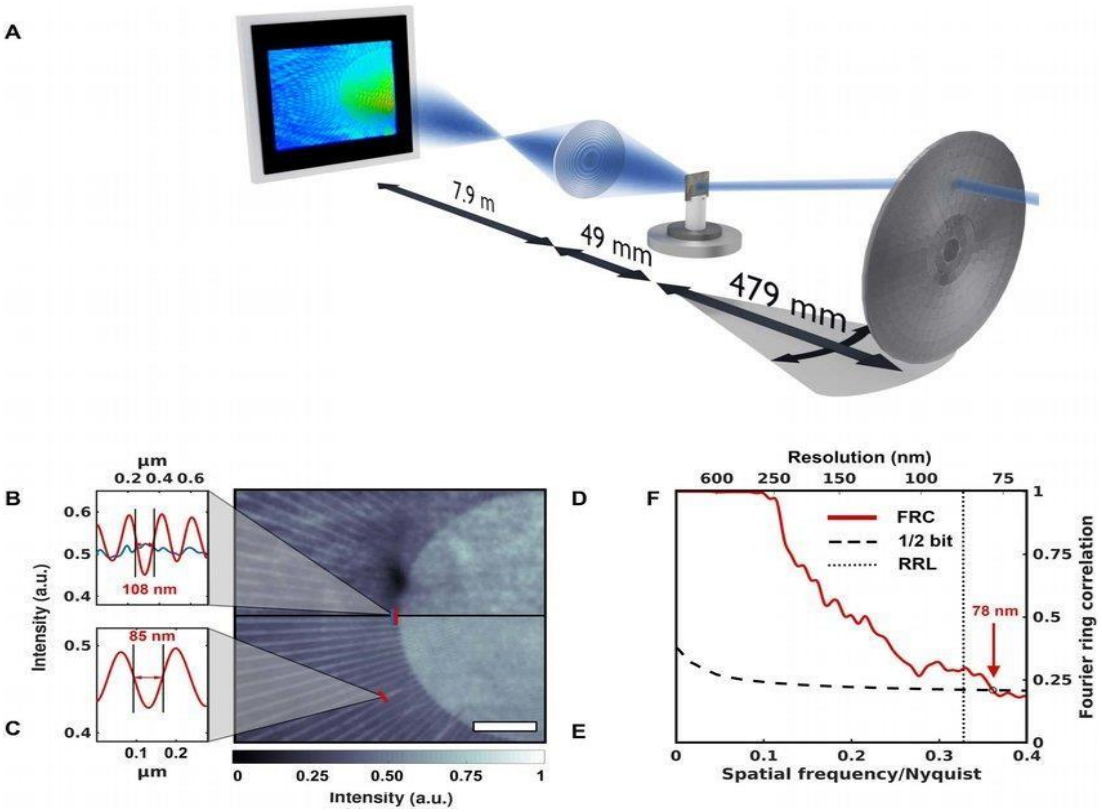
Cross correlation of similar images for co-alignment;

X-ray crystallography to reconstruct a crystal structure from its diffraction pattern;

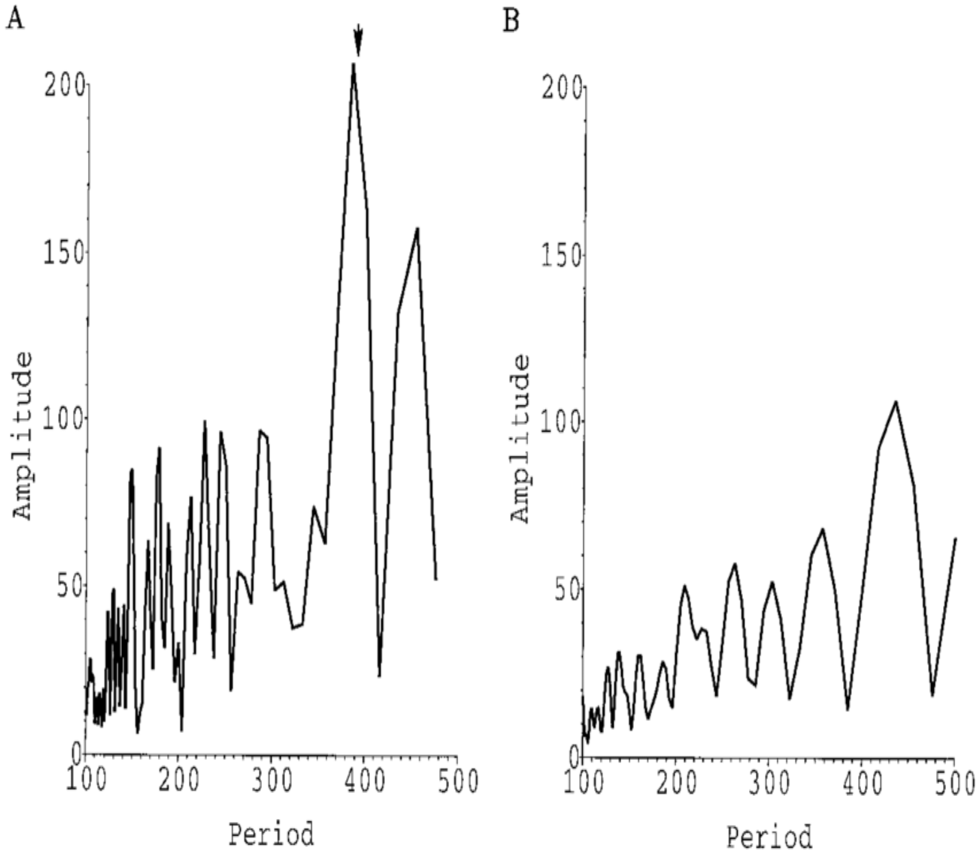
Fourier transform ion cyclotron resonance mass spectrometry to work out the mass of ions from the frequency of cyclotron motion during a magnetic field;



X-Ray-Diffraction-analysis



X-ray-Fourier-ptychography-by-scanning



Conclusion:

After knowing the basic theory behind Dirichlet characters, it results almost

natural tonuse DFT to build a relation between them and additive characters. In the beginning of this
work, Terras' book was the principal source of topics to be developed. The theory from chapters 2 and 3 follows the same methodology and contains similar results as in the book. However, none of the contents of chapter 4 can be found in it. More details about multiplicative number theory can be found in [9], [6], [7] and [10]. As an historical note, Dirichlet characters were first defined by Peter Gustav Lejeune Dirichlet in 1831. Back then, he was looking for functions from Z to C having the properties we stated in chapter 4. The most important use of Dirichlet characters is to define Dirichlet L- functions, which are common elements in books and papers about number theory nowadays. Thus, possible further readings may include topics in analytic number theory dealing with L-functions. Furthermore, in chapter 5 we gave an application of DFT. Even though graph theory may seldom classify as pure mathematics, the number of industrial applications of it has been growing since its beginnings. This justifies the fact that there are
more and more people doing research about it. In this

work we presented very particular results that can be used to solve certain problems about graphs, but DFT has several other applications such as error correcting codes, compressing algorithms or even problems from physics and chemistry (some of them can be found in. All the theory developed in this work is sufficiently straightforward to be used either in analytic or algebraic number theory (doing a brief classification of topics in number theory). Nonetheless, the tools and ideas presented, DFT to be precise, are powerful enough that one can find them in much deeper notes or even research papers.

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With applications to representation theory and geometry.