ELEMENTS OF POINT SET TOPOLOGY

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ABSTRACT

Point set topology are terse introduction to the topological concepts used in economic theory. Topology is a basic mathematical field that deals with geometric properties, continuity, and boundary in relation to subspaces. Tynchonoff's theorem is classified as of the topology theorem.

KEYWORDS

Topology, topological space, open sets, closed sets, compact sets, open ball, closed ball

INTRODUCTION

Elements of point set topology is the branch of toplogy that deals with the basic set-theoritic definitions and constrctions used in topology. Another name of point set topology is algebraic topology, general topology, set theoritic topology.

BASIC DEFINITION:

DEFINITION

A topology on a group X may be a collection τ of subsets of X having the subsequent properties

(i) ∅ and x are in τ

(ii) The union of the weather of any subcollection of τ is in τ

(iii) The intersection of the weather of any finite subcollection of τ is in τ

EXAMPLES

$X = \{1,2,3,4\}$,

$\tau = \{\{\ \}, \{2\}, \{1,2\}, \{1,2,3\}, \{1,2,3,4\}\}$

τ is a topology on X

REMARKS

A topology τ is an open sets

DEFINITION

If X is any set, the collection of all subset of X is a topology on x it is called the discrete

topology

DEFINITION

If X is any set, the collection of X consisting of X and ∅ only is a topology on X it is called the indiscrete topology (or)trivial topology

DEFINITION

Let X be a set, τ_f be the collection of all subsets U of X. such that X-U either is finite or is

all of X.Then τ_f is topology on X, called the finite complement topology.

DEFINITION

Let X be any infinite set. Define a topology on X by A=τ is X-A os finite or

A=∅. This is called the Zariski topology(0r) co-finite topology.

CLOSED SET, ADHERENT POINT AND ACCUMULATE POINTS

DEFINITION

Let S be a subset of R^n and Let X be a point in R^n,X not necessarily in S. Then X is said

to be adherent to S if everyn-ball B(X)

EXAMPLE

If S is a subset of R which bounded above, then supremum is adherent to S.

DEFINITION

If S⊆ R^n and xϵR^n, then s is called an accumulation point of S if every n-ball B(x)

contains atleast one point of S distinct from X

EXAMPLE

The set of rational numbers has every number is an Accumulation point

DEFINITION

A set S in R^n is named closed if its complement R^n-S is open.

THEOREM

In any mathematical space X, each closed sphere may be a closed set.

PROOF

Let X be a metric space.

Assume that,

$S_r[x_0]'$ is non-empty

Let $S_r[x_0]$ be a closed sphere in X.

If $S_r[x_0]'$ is open and it is empty.

Let x be a point in $S_r[x_0]'$

Since, d(x,x_0) $> r$, r_1=d(x,x_0)-r is a positive real number, r_1 as the radius of an open sphere $S_{r1}(x)$ centered on x.

We show that,

$S_r[x_0]'$ is open

$S_{r1}(x) \subseteq S_r[x_0]'$

Let y be a point in $S_{r1}(x)$

$$d(y,x) < r_1$$

$$d(x,x_0) \leq d(x_0,y) + d(y,x)$$

$$d(y,x_0) \geq d(x,x_0) - d(y,x)$$

$$> d(x,x_0) - r_1$$

$$= d(x,x_0) - [d(x,x_0) - r]$$

$$d(y,x_0) = r$$

Since, y is in $S_r[x_0]'$

$S_r(x_0)$ be a closed.

Hence, proved.

THEOREM

The intersection of a finite collection of open sets is open.

PROOF

Let S denote their intersection

$S = \bigcap_{K=1}^{M} A_k$

Then each A_k is open

Assume that,

x ∈ S

If x is empty.

There is nothing to prove,

x ∈ A_k, for every K = 1,2,…,n

Hence, there is an open n - ball B(x;r_k) ⊆ A_k

Let r be the smallest positive numbers $r_1, r_2, \ldots r_m$

Then, x∈B(x;r)

Since, B(x;r) ⊆ S

X is an interior point. S is open

Hence, their intersection of a finite collection of open sets is also open

Hence, proved.

DEFINITION

Let X be a non-empty set. A function d:$X \times X \to \mathbb{R}$ is said to be metric space on X if it satisfies the following conditions

I. d(x,y)≥0 with d(x,y)=0, $\forall x, y \epsilon X$
II. d(x,y)=d(y,x), $\forall x, y \epsilon X$
III. d(x,y)≤d(x,z)+d(z,y), $\forall x, y, z \epsilon X$

Note:

Let (X,d) is a metric space on X, X is a non-empty set.

CONCLUSION:

The notions of groups and functions in topological spaces, ideal mathematical spaces, minimal spaces and ideal minimal spaces are extensively developed and utilized in many engineering problems, information systems, high-energy physics , computational topology and mathematical physics.

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