DOMINATION AND DOMATIC NUMBER OF A GRAPH

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ABSTRACT

A dominating set of a graph G = (V, E) may be a subset D of V. such every vertex not in D is adjacent to some vertex in D. The domatic number d(G) of G is that the maximum positive integer K. such V are often partitioned into K pairwise disjoint dominating sets. the aim of this paper is to review the domatic numbers and domination numbers of a graphs that are obtained from small graphs by performing graph operations.

KEYWORDS

Domatic number,Domatic partition,Domination Number

INTRODUCTION

Graph Theory is one among the foremost flourishing branches of Mathematics with applications to a good sort of subjects.Graphs are met everywhere under different names. Structures in engineering , Networks in EE , Communication Structures and Organizational Structures in Sociology and Economics, Molecular Structures in Chemistry, Roadmaps or Electricity distribution Networks then,

This projects deals with the Domination and Domatic Number of a graph.

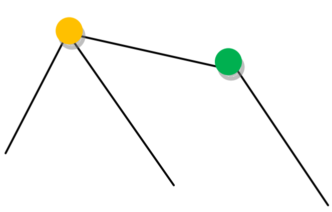
Graph is one among the foremost important and officient branches in mathematics. There are applications of graphs Inoory to some area of physics chemistry, science, communication networks.

Graph theory may be a major a part of discrete mathematics. Among the various aspects of graph theory. Decomposition may be a vast area of research. the aim of the study of decomposition of a graph G is to seek out that number n that the graph G is to seek out that number are often distorted into n copies of a hard and fast graph or n different graphs.

DOMATIC NUMBER

The Domatic number is that the maximum size of a domaticpartition, that's the utmost number of disjoint dominating sets.

Example



The graph in the figure has domatic number 3

DOMATIC PARTITION

Let G = (V, E) be a graph. A domatic partition is a partition of V into dominating sets and the domatic number d(G) is the largest number of sets in a domatic partition and its is denoted by d(G). From the definition it is clear that for any graph G, d(G) ≤ δ(G) + 1.

DOMINATION NUMBER

The domination number $\gamma(G)$ of a graph G, denoted $\gamma(G)$ is the minimum size of a dominating set of vertices in G, ie., the size of a minimum dominating set.

THEOREM

Let G be a graph such that both G and $\bar{G}$ have no isolated verticesand $d + \bar{d} = p$. Then the following hold.

(i) p is even

(ii) $\gamma = \bar{\gamma} = 2$

(iii)$d + \bar{d} = \frac{p}{2}$

(iv)$deg_G v \frac{p}{2} - or \frac{p}{2} for\ all\ v \in V.$

Proof

Since G and $\bar{G}$ have no isolated vertices, $\gamma \geq 2$ and $\bar{\gamma} \geq 2$ so that $d \leq \frac{p}{2} and\ \bar{d} \leq \frac{p}{2}$. Since $d + \bar{d} = \frac{p}{2}$, it follows that p is even, $\gamma = \bar{\gamma} = 2$ and $d + \bar{d} = \frac{p}{2}$.

Now let p = 2n and {D_1, D_2,…………,D_n} be a domatic partition of G. Then $|D_i|$ = 2 for each i and every vertex of D_i is adjacent to at least one vertex of each $D_j(j \neq i)$ so that degree of each vertex is at least n – 1. Similarly degree of each vertex is at least n – 1 in $\bar{G}$. Hence degree of any vertex is either n – 1 or n.

COROLLORY

Let G be any graph such that G and $\bar{G}$ have no isolated vertices and $\delta(G) = 1$. Then $d + \bar{d} = p$ if and only if $G \cong P_4$or $2K_2$.

Proof:

Let $d + \bar{d} = p$; By theorem 2.1 degree of any vertex is either $\frac{p}{2} - 1$ or $\frac{p}{2}$.

Since $\delta = 1$, it follows that p = 4 and G is isomorphic to P_4 or $2K_2$.

The converse is obvious.

A graph G is said to be domatically fullif d(G) = δ(G) + 1.

A graph G is said to be strong-weak domaticallyfullif d^*(G) = δ(G) + 1.

THEOREM

Let G be a disconnected graph with no isolated vertices. Then $\gamma(G) = \gamma(\bar{G})$ if and only if G has exactly two components each with domination number one.

Proof

Given that $\gamma(G) = \gamma(\bar{G})$. Let $\gamma(G) = \gamma(\bar{G})$ = k(Say).

To prove G has exactly two components with domination number one.

Suppose G has k components. Then $\gamma(G) = k$

In G, let u_1, u_2…………..u_k be the vertex in the components g_1, g_2……..g_k which is connected with the remaining vertex in g_1, g_2……..g_k.

In $\bar{G}$, u_1 is connected with the vertex in the components g_1, g_2……..g_k.

But it is not connected with remaining vertex in g_1.

Similarly, in $\bar{G}$, u_2 is connected with the vertex in the components g_1, g_2……..g_k. But it is not connected with remaining vertex in g_2.

$\gamma(\bar{G})$=2

But $\gamma(G) = \gamma(\bar{G})$ = k = 2

K = 2

Therefore, G has an exactly two components with domination number one.

Conversely, assume that G has an exactly two components with domination number one.

To prove $\gamma(G) = \gamma(\bar{G})$, we use the statement "G has an exactly two components with domination number one".

$$\gamma(G) = \gamma(g_1) + \gamma(g_2)$$

$$= 1 + 1$$

$$\gamma(G) = 2$$

G has exactly two components g_1 and g_2 with domination number one.

Let u be the vertex in g_1 which is connected with the remaining vertex in g_1 and let D_1 = {u} be the dominating set in g_1.

Similarly, let v be the vertex in g_2 which is connected with the remaining vertex in g_2.

Let D_2 = {v} be the dominating set in g_2.

In $\bar{G}$ u is connected with all the vertices in g_2.

But it is not connected with the remaining vertex in g_1.

Similarly v is connected with all the vertices in g_1.

But it is not connected with the remaining vertex in g_2.

{u, v} in $\bar{G}$ is a dominating set.

$\gamma(\bar{G})$ = 2

Hence we proved.

THEOREM

A tree T is a strong – weak domatically full if and only if T has no vertex x satisfying the following conditions.

(i) X is a nonsupport

(ii) X is adjacent only to supports

(iii)deg x >deg y for each y ∈ N (x)

Proof

Sufficient part :

Let T has no vertex x satisfying the following conditions.

(i) X is a non support

(ii) X is adjacent only to supports

(iii)deg x >deg y for each y ∈ N (x)

Then by the theorem 1.4 an sd-set D of T and V – D as a wd-set of T.

T has two adjoint dominating set.

$d^{*}(T) = 2$

$d^{*} (T) = 1 + 1 = \delta + 1$

Necessary part

Let tree T be a strong – weak domatically full.

i.e) $d^{*} (T) = \delta + 1$

$d^{*}(T) = 2$

D as asd-set of T. V – D as a wd-set of T.

Then by theorem 7 T has no vertex x satisfying these three conditions.

CONCULSION

The present century has witnessed a steady development of graph theory which in the left ten to twenty years has blossomed out into a new period of intense activity and many of them together to obtain a new type of graph and its properties.

In this project Domination and domatic number of a graph strong – weak domatic numbers of graphs have been discussed.

We can extend the project in some different kinds of domatic number of a graph like point – set domatic number of a graph, connected domatic number of a graph and total domatic number of a graph.

REFERENCES

1) S. Arumugam and S.Ramachandran, Invitation to Graph Theory, New Gamma Publishing House.
2) F. Harary, Graph Theory, Addision – Wesley, Reading Mass. (1969)
3) V.Swaminathan and P.ThangaRaju, “Strong – weak Domatic Numbers of Graphs” Indian Journal of Pure and Applied Mathematics. 33(12) : 1841 – 1846December 2002.