

**DUAL – HESITANT FUZZY TRANSPORTATION PROBLEM**  
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**ABSTRACT**

So on handle inexact, tentative or partial information and knowledge circumstances in factual-life operational investigate predicaments dual-hesitant fuzzy set is applied. during this work a totally unique method called Allocation Table Method (ATM) for solving dual-hesitant fuzzy transportation problem is introduced. This method is explained with a numerical example and thus the result obtained through this method is compared with the prevailing method (VAM). This proposed method gives an optimum solution as compared with other method.

Keywords: ATM, dual-hesitant fuzzy, dual-hesitant fuzzy numbers. Score function, transportation problem, VAM.

**INTRODUCTION**

Transportation may be a strapping pact to congregate the tackle of the way to provide the products to the clients in extra skillful techniques. They assure the proficient evolution and sagacious openness of raw materials and completed goods. Hitchcock [6] developed the transportation problem (TP) in 1941 , Charnes et al [1] (stepping stone technique) in 1953 and Dantzig [2] (primal simplex transportation) in 1963,have studied transportation problems and provided various techniques to unravel the TP.

All the constraints of the TPs won't be recognized explicitly because of insurmountable factors in extant relevance's. By selecting a variate from a probability distribution this type of vague information can't be represented clearly. To handle this instance Zadeh in 1965 [16] introduced the fuzzy numbers. Zimmermann in 1978 [18] projected that the elucidations acquired by fuzzy applied math are forever dexterous. a completely unique category of fuzziness, to be precise, dual-hesitant fuzzy number is introduced to recuperate the complexity in realistic circumstances, which put ups the circumstances carefully.

The notion of a Hesitant Fuzzy Set (HFS) was primary instigated by Torra and Narukawa in 2009, 2010 [14, 15]. Dual-Hesitant Fuzzy Sets (DHFSs), which are an expansion of HFSs that cover fuzzy sets, intuitionistic fuzzy sets, HFSs and fuzzy multi-sets as special cases ,which was presented by Zhu et al in 2012 [17]. Singh in 2014 [13] proposed a study on deciphering assignment problems with DHFSs. To recover a replacement class of fuzziness, namely, dual-hesitant fuzzy number, which accommodates things nicely and provides a much better result by using fuzziness. A dual-hesitant fuzzy transportation problem with restrictions was proposed by Gurupada Maity et al in 2019 [4]. Saad and Abbas in 2003 [12] extended an algorithm for locating the elucidation for the TPs in fuzzy environment. Fuzzy zero method was projected by Pandian & Natrajan in 2010

[10,11] to unravel TPs. Hajjari & Abbasbandy in 2011 [5] projected a promoter operator for defuzzification methods with method of magnitude. Mollah Mesbahuddin Ahmed et al in 2016 [9] have proposed Incessant Allocation Method for Solving Transportation Problems. Md Sharif Uddin et al in 2016 [8] have provided the Allocation Table Method for Solving Transportation Maximization Problems. Darunee Hunwisaiand Poom Kumam in 2017 [3] solved a fuzzy transportation problem via Robust ranking technique and ATM. By using method of magnitude fuzzy transportation problem is solved by Krishna Prabha et al in 2018[7].

In this research work we've considered the same example proposed by Gurupada Maity et al for solving a dual-hesitant fuzzy transportation problem which was solved by using VAM, which we've proposed ATM method to unravel the same example and compared the results.

In this paper a dual-hesitant fuzzy transportation problem is deciphered by ATM method. A preface to fuzzy sets and DHFs are presented in section 2. In section 3 an algorithm for solving the matter is proposed. In section 4, a numerical example is illustrated. Conclusion of the work is given in section 5.

## II. PREFACE

### 2.1 Hesitant fuzzy sets (HFS)

#### Definition 2.1 (Torra [14, 15]).

A HFS  $H$  on  $X$  is defined in terms of a function  $h(x)$  that returns a subset of values within the interval  $[0, 1]$  once it's applied to  $X$ , i.e., a component of its power set:  $h: X \rightarrow \mathcal{P}([0,1])$ . Thereafter, Xu and Xia (2011) described the definition of HFS during a compact form by including the mathematical representation of a HFS.

#### Definition 2.2 (Xu and Xia [17]).

A HFS is stated mathematically within the subsequent way:  $H = \{(x_i, h(x_i)) : x_i \in X\}$  where  $h(x_i)$  could also be a group of several different values within the interval  $[0, 1]$  for each  $x_i \in X$ , which denotes the possible membership degree of the element  $x_i \in X$  within the set  $H$ . within the usual sense, each member of  $h(x_i)$  is named a Hesitant Fuzzy Element (HFE), denoted by  $h_i$ .

### 2.2 Score of a HFS [4]

#### Definition 2.3:

Let us consider a HFS  $H = \{(x_i, h(x_i)) : x_i \in X\}$ , where  $X = \{x_1, x_2, x_3, \dots, x_n\}$  is a finite set. For a HFE  $h_i$  in  $H$ , we define a score function, denoted as  $s(h_i)$  and defined as follows:

$$s(h_i) = \frac{\sum_{j=1}^k (h(x_j))}{k} \quad (i= 1,2,3,\dots,n)$$

Here, the important number  $s(h_i)$  is known as the score of  $h_i$ , where  $k$  is that the amount of elements in  $h_i$ .

Let  $h_1$  and  $h_2$  be two HFEs.

Case 1: If  $s(h_1) > s(h_2)$ , then  $h_1$  is known as superior to  $h_2$ , denoted by  $h_1 > h_2$ .

Case 2: If  $s(h_1) < s(h_2)$ , then  $h_1$  is known as inferior to  $h_2$ , denoted by  $h_1 < h_2$ .

Case.3: If  $s(h_1) = s(h_2)$ , then  $h_1$  is known as indifferent from  $h_2$ , denoted by  $h_1 \sim h_2$ .

### 2.3 Dual hesitant fuzzy sets [4]

#### Definition 2.4

Let  $X$  be a hard and fast set; then a DHFS  $D$  on  $X$  is defined as follows:  $D = \{(x, h(x), g(x)) : x \in X\}$  where  $h(x)$  and  $g(x)$  are mappings that take set-values in  $[0, 1]$ ; they're denoted because the possible membership degree and non-membership degree of any element  $x \in X$ , to the set  $D$ , respectively, with the conditions  $0 \leq h(x), g(x) \leq 1, 0 \leq h(x) + g(x) \leq 1$ , for any  $h(x) \in h(x); g(x) \in g(x)$ .

A Dual-Hesitant Fuzzy element (DHFE) is known because the pair  $d(x) = (h(x), g(x))$ , and it's denoted within the functional form as  $d = (h, g)$ .

#### 2.5 Arithmetic operations on DHFEs [4]

Let  $d_1 = (h_1, g_1)$  and  $d_2 = (h_2, g_2)$  represent two DHFEs; then addition and subtraction is given by,

Addition,  
 $d_1 \oplus d_2 =$

Subtraction,  
 $d_1 \ominus d_2 =$

#### 2.6 Ranking of dual hesitant fuzzy sets [4]

Let  $D = \{(x, h(x), g(x)) : x \in X\}$  be a DHFS, where  $X = \{x_1, x_2, x_3, \dots, x_n\}$  and  $d = (h, g)$  be a DHFE. We define a score function  $S_d$  on the DHFS, represented as follows:

Let  $d_1$  and  $d_2$  be any two DHFSs.

With reference to a given score function, Zhu et al (2012) defined order relations as follows:

Case 1: If  $S_{d_1} > S_{d_2}$ , then  $d_1$  is named superior to  $d_2$ , denoted by  $d_1 > d_2$ .

Case 2: If  $S_{d_1} < S_{d_2}$ , then  $h_1$  is named inferior to  $h_2$ , denoted by  $d_1 < d_2$ .

Case 3: If  $S_{d_1} = S_{d_2}$ , then  $h_1$  is named indifferent from  $h_2$ , denoted by  $d_1 \sim d_2$

### III PROBLEM FORMULATIONS [7]

The balanced fuzzy transportation problem, during which a choice maker is uncertain about the precise values of transportation cost, availability and demand, could also be formulated as follows:

$$\text{minimize } \sum_{i=1}^p \sum_{j=1}^q c_{ij} * x_{ij}$$

Subject to  $\sum_{j=1}^q x_{ij} = \tilde{a}_i, i = 1, 2, 3, \dots, p$

$$\sum_{i=1}^p x_{ij} = b_j, j = 1, 2, 3, \dots, q$$

$$\sum_{i=1}^p a_i = \sum_{j=1}^q b_j$$

$x_{ij}$  could also be a non-negative trapezoidal fuzzy number, Where  $p$  = total number of sources  $q$  = total number of destinations  $a_i$  = the fuzzy availability of the merchandise at  $i$ th source  $b_j$  = the fuzzy demand of the merchandise at  $j$ th destination  $c_{ij}$  = the fuzzy transportation cost for unit quantity of the merchandise from  $i$ th source to  $j$ th destination

$x_{ij}$  = the fuzzy quantity of the merchandise that need to be transported from  $i$ th source to  $j$ th destination to attenuate the whole fuzzy transportation cost. we use the sole kind of notation for dual-hesitant fuzzy cost as =  $(h_{ij}, g_{ij})$  within the rest of our discussion. Hence, the dual-hesitant fuzzy cost is introduced so on style the mathematical model of the TP; we name it as aDHFTPR which we put this within the subsequent way:

$$\text{optimize } \sum_{i=1}^p \sum_{j=1}^q \tilde{c}_{ij} * x_{ij}$$

Subject to  $\sum_{j=1}^q x_{ij} = \tilde{a}_i, i = 1, 2, 3, \dots, p$

$$\sum_{i=1}^p x_{ij} = b_j, j = 1, 2, 3, \dots, q$$

$$\sum_{i=1}^p a_i = \sum_{j=1}^q b_j$$

In fact, the objective function ( $z$ ) in problem is designed based on dual-hesitant fuzzy costs  $\tilde{c}_{ij}$ .

#### IV. ALLOCATION TABLE METHOD (ATM) [7]

ATM for solving transportation problems to attenuate the worth is illustrated below.

Step-1: create a Dual-Hesitant Fuzzy Transportation Table (DHFTT) from the required transportation problem.

Step-2: confirm if the TP is balanced or not, if not, make it balanced.

Step-3: Pick the smallest amount Odd Cost (LOC) from all the value cells of DHFTT.

If there's no odd cost within the cost cells of the DHFTT, keep it up dividing all the value cells by 2 (two) till accessing least an odd cost within the cost cells.

Step-4: Construct a replacement table which is to be identified as allocation table (AT) by keeping the LOC within the respective cost cell/cells because it was/were, and subtract selected LOC only from each of the odd cost valued cells of the DHFTT. Now all the cell values are to be called as Allocation Cell Value (ACV) in AT.

Step-5: Categorize the utmost ACV and allot minimum of supply/demand at the place of selected

ACV within the AT. just in case of same ACVs, select the ACV where maximum allocation are often made. Again just just in case of same allocation within the ACVs, choose the

utmost cost cell which is just like the value cells of DHFTT formed in Step-1 (i.e. this maximum cost cell is to be acknowledged from the DHFTT which is formed in Step 1). Again if the worth cells and thus the allotments are equal, in such case choose the nearer cell to the minimum of demand/supply which is to be allocated. Now if demand is satisfied delete the column and if it's supply delete the row.

Step-6: Re iterate Step 5 until the demand and supply are exhausted.

Step-7: Now shift this allocation to the primary DHFTT.

Step-8: To conclude, compute the whole profit of the DHFTT. This calculation is that the sum of the merchandise of cost and resultant allocated value of the DHFTT

## V. CONCLUSION

In this paper, the ATM is applied during a Dual-Hesitant Fuzzy Transportation problem. we've compared our results to point out that our approach yields a far better optimum solution then the prevailing approach[4]. this system are often used to solve all types of Dual-Hesitant Fuzzy Transportation problems. Consequently this scheme are often utilized to resolve the real-life problems and provide chain management.

## REFERENCE

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