AN OVER VIEW ON HYPER FUZZY SUBGROUPS

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ABSTRACT

Now we define hyper fuzzy set as a generalization of fuzzy sets and interval valued fuzzy sets. Then we introduce the notion of hyper fuzzy subgroups and study some of its important properties.

INTRODUCTION

In this project we are studied about the concept of hyper fuzzy sets where the values of the membership function are assumed to be a subsets of [0,1] which is a generalized of fuzzy sets and interval-valued fuzzy sets.

Hyper fuzzy subgroup is defined and a few of its basic properties are studied.

We also analysis about hyper fuzzy left and right cosets and also define hyper fuzzy normal subgroups and study a few of its properties.

KEYWORDS

Fuzzy ,Sub Group ,Hyper fuzzy sub group

DEFINITION

Let X be a set. Then mapping $\hat{\mu} : X \rightarrow P^{*}([0,1])$ is called a hyper fuzzy subset of X where $P^*([0,1])$ denotes the set of all non empty subset of [0,1].

DEFINITION

Let X be a non empty set and $\hat{\mu}, \hat{\nu}$ be two hyper fuzzy subset of X. then intersection of $\hat{\mu}$ and $\hat{\nu}$ is denote $\hat{\mu} \cap \hat{\nu}$ and defined by

(i) $(\hat{\mu} \cap \hat{\nu})(X) = \{\min\{p,q\} : p \in \hat{\mu}(X), q \in \hat{\nu}(X)\}$ for all $x \in X$

Then union $\hat{\mu}$ and $\hat{\nu}$ is denoted by $\hat{\mu} \cup \hat{\nu}$ and defined by

(ii) $(\hat{\mu} \cup \hat{\nu})(X) = \{\max\{p,q\} : p \in \hat{\mu}(X), q \in \hat{\nu}(X)\}$ for all $x \in X$

DEFINITION

Let X be a groupoid i.e) a set which is closed under a binary relation denoted multiplicatively.

A mapping $\hat{\mu} : X \to P^*([0,1])$ is called a hyper fuzzy subgroupoid if $\forall\, x, y \in X$ following conditions hold.

(iii) $\inf \hat{\mu}(xy) \geq \min\{\inf \hat{\mu}(x), \inf \hat{\mu}(y)\}$

(iv) $\sup \hat{\mu}(xy) \geq \min\{\sup \hat{\mu}(x), \sup \hat{\mu}(y)\}$

DEFINITION

Let G be a group. A mapping $\hat{\mu} : X \to P^*([0,1])$ is called a hyper fuzzy subgroupoid if $\forall\ x, y \in X$ following conditions hold.

(v) $\inf \hat{\mu}(xy) \geq \min\{\inf \hat{\mu}(x), \inf \hat{\mu}(y)\}$ (vi)

$\sup \hat{\mu}(xy) \geq \min\{\sup \hat{\mu}(x), \sup \hat{\mu}(y)\}$ (vii)

$\inf \hat{\mu}(x^{-1}) \geq \inf \hat{\mu}\ (x)$

(viii) $\sup \hat{\mu}(x^{-1}) \geq \sup \hat{\mu}\ (x)$

PROPOSITION

If $\hat{\mu}$ is a hyper fuzzy subgroup of G having the identity e, then for all x ϵ X.

(i) $\inf \hat{\mu}(x^{-1}) \geq \inf \hat{\mu}\ (x)$ and $\sup \hat{\mu}(x^{-1}) \geq \sup \hat{\mu}\ (x)$

(ii) $\inf \hat{\mu}(e) \geq \inf \hat{\mu}\ (x)$ and $\sup \hat{\mu}(e) \geq \sup \hat{\mu}(x)$

PROOF

(i) As $\hat{\mu}$ is a hyper fuzzy subgroups of group G then

$\inf \hat{\mu}(x^{-1}) \geq \inf \hat{\mu}\ (x)\ \forall\ x \in G$

again,

$$\inf \hat{\mu}(x) = \inf \hat{\mu}(x^{-1}) \geq \inf(x^{-1})$$

so,

$$\inf \hat{\mu}(x^{-1}) = \inf \hat{\mu}(x)$$

similarly,

we can prove that

$$\sup \hat{\mu}(x^{-1}) = \sup \hat{\mu}(x)$$

(ii) $$\inf \hat{\mu}(e) = \inf \hat{\mu}(xx^{-1})$$

$$\geq \min\{\inf \hat{\mu}(x), \inf \hat{\mu}(x^{-1})\}$$

$$= \inf \hat{\mu}(x)$$

And $$\sup \hat{\mu}(e) = \sup \hat{\mu}(xx^{-1})$$

$$\geq \min\{\sup \hat{\mu}(x), \sup \hat{\mu}(x^{-1})\}$$

$$= \sup \hat{\mu}(x)$$

Hence the proposition is proved.

PROPOSITION

A hyper fuzzy subset $\hat{\mu}$ of group G is a hyper fuzzy subgroup iff for all x,y ϵ G following are hold.

(i) $\inf \hat{\mu}(xy^{-1}) \geq \min\{\inf \hat{\mu}(x), \inf \hat{\mu}(y)\}$

(ii) $\sup \hat{\mu}(xy^{-1}) \geq \min\{\sup \hat{\mu}(x), \sup \hat{\mu}(y)\}$

PROOF

At first let $\hat{\mu}$ be a hyper fuzzy subgroup of G and x,y ϵ G.

then

$$\inf \hat{\mu}(xy^{-1}) \geq \min\{\inf \hat{\mu}(x), \inf \hat{\mu}(y^{-1})\}$$

$$= \min\{\inf \hat{\mu}(x), \inf \hat{\mu}(y)\} \text{ and}$$

$$\sup \hat{\mu}(xy^{-1}) \geq \min\{\sup \hat{\mu}(x), \sup \hat{\mu}(y^{-1})\}$$

$$= \min\{\sup \hat{\mu}(x), \sup \hat{\mu}(y)\}$$

Conversely,

Let be a hyper fuzzy subset of G and given condition hold.

Then for all x ϵ G we have

$$\inf \hat{\mu}(e) = \inf \hat{\mu}(xx^{-1})$$

$$\geq \min\{\inf \hat{\mu}(x) , \inf \hat{\mu}(x)$$

$$= \inf \hat{\mu}(x) - --\rightarrow (1) \text{ sup}$$

$$\hat{\mu}(e) = \sup \hat{\mu}(xx^{-1})$$

$$\geq \min\{\sup \hat{\mu}(x) , \sup \hat{\mu}(x)$$

$$= \sup \hat{\mu}(x) - --\rightarrow (2)$$

So,

$$\inf \hat{\mu}(x^{-1}) = \inf \hat{\mu}(ex^{-1})$$

$$\geq \min\{\inf \hat{\mu}(e), \inf \hat{\mu}(x)$$

$$= \inf \hat{\mu}(x) - --\rightarrow by\ (1)$$

By

$$\sup \hat{\mu}(x^{-1}) = \sup \hat{\mu}(ex^{-1})$$

$$\geq \min\{\sup \hat{\mu}(x), \sup \hat{\mu}(x)$$

$$= \sup \hat{\mu}(x) - --\rightarrow by\ (2)$$

Again,

$$\inf \hat{\mu}\ (xy) \geq \min\{\inf \hat{\mu}(x), \inf \hat{\mu}(y^{-1})\}$$

Using given condition

$$\geq \min\{\inf \hat{\mu}(x), \inf \hat{\mu}(y)\}$$

And

$$\sup \hat{\mu}\ (xy) \geq \min\{\sup \hat{\mu}(x), \sup \hat{\mu}(y^{-1})\}$$

using given condition

$\geq \min\{\sup \hat{\mu}(x), \sup \hat{\mu}(y)\}$ by above Hence

$\hat{\mu}$ hyper fuzzy subgroup of G.

Hence the proposition is proved.

PROPOSITION

Intersection of any two hyper fuzzy subgroups of a group G is called also a hyper fuzzy subgroup of G.

PROOF

Let $\hat{\mu}$ and $\hat{\nu}$ be two hyper fuzzy subgroups of a group G.

$x, y \in G$.

Then

$$\inf(\hat{\mu} \cap \hat{\nu})(xy^{-1}) = \min\{\inf \hat{\mu}(xy^{-1}), \inf \hat{\nu}(xy^{-1})$$

$$= \{\min \{\min\{\inf \hat{\mu}(x), \inf \hat{\mu}(y)\ ,\ min\ \{\inf \hat{\nu}(x), \inf \hat{\nu}(y)\}\}$$

$$= \{\min \{\min\{\inf \hat{\mu}(x), \inf \hat{\nu}(y)\ ,\ min\ \{\inf \hat{\mu}(x), \inf \hat{\nu}(y)\}\}$$

$$= \min\{\inf(\hat{\mu} \cap \hat{\nu})(x)\ , \inf(\hat{\mu} \cap \hat{\nu})(y) \ldots \ldots (3)$$

Again

$$\sup(\hat{\mu} \cap \hat{\nu})(xy^{-1}) = \min\{\sup \hat{\mu}(xy^{-1}), \sup \hat{\nu}(xy^{-1})\}$$

$$= \{\min \{\min\{\sup \hat{\mu}(x), \sup \hat{\mu}(y)\ ,\ min\ \{\sup \hat{\nu}(x), \sup \hat{\nu}(y)\}\}$$

$$= \{\min \{\min\{\sup \hat{\mu}(x), \sup \hat{\nu}(y)\ ,\ min\ \{\sup \hat{\mu}(x), \sup \hat{\nu}(y)\}\}$$

$= \min\{\sup(\hat{\mu} \cap \hat{\vartheta})(x)\ , \sup(\hat{\mu} \cap \hat{\vartheta})(y) \ldots\ldots (4)$

Hence (3) and (4) using we say that $\hat{\mu} \cap \hat{v}$is a hyper fuzzy subgroup of G. Hence proposition is proved.

CONCLUSION

The concept of fuzzy set is very simple and easy to understand. In a short span of time. We have done only a little drops in this field. In this project we have done the basic definitions of fuzzy concepts on normal and hyper subgroups.

The above study has helped me to derive some equivalent conditions for each fuzzy subgroups. It has helped me to acquire some that basic knowledge of fuzzy.

As an application we have highlighted the role of the fundamental concept of fuzzy sets helped to various fields.

The fundamental concept of fuzzy sets play a prominent role in mathematics with wide application in many other branches such as theoretical physics, computer science, control engineering information science, coding theory , graph theory real analysis measure theory etc,…

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