AN OVER VIEW OF VASSILIEV INVARIENTS

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ABSTRACT

Mathematical studies of knots began within the $19th$century with Carl Friedrich Gauss, who defined the linking integral (silver 2006). within the 1860s, Lord Kelvin's theory that atoms were knots within the aether led to Peter Guthrie Tait's creation of the primary knot tables for complete classification.

INTRODUCTION

Early modern, knots were studied from a mathematical viewpoint by Carl Friedrich Gauss, Who in 1833 developed the Gauss linking integral for computing the linking number of two knots..He formulated what are now referred to as the Tait conjectures on alternating knots. (The conjectures were proved within the 1990s.

Mathematical studies of knots began within the $19th$century with Carl Friedrich Gauss, who defined the linking integral (silver 2006). within the 1860s, Lord Kelvin's theory that atoms were knots within the aether led to Peter Guthrie Tait's creation of the primary knot tables for complete classification.

While tabulation remains a crucial task, today's researchers have a good sort of background and goal.

In the last 30 years knot theory has also become a tool in applied math . Chemists and biologists use knot theory to know .

For example, chirality of molecules and therefore the actions of enzymes on DNA. Knot theory in mathematics the study of closed curves in three dimensions , and their possible deformations without one part cutting through another.

Knots could also be considered formed by interlacing and looping a bit of string in any fashion then joining the ends.

KEY WORDS

Vassiliev invariants, Polynomial, Topological Space

Definition:

Let us assume $V0$ is a few numerical knot invariant. ie) to every knot K,$V0$assigns a real number . Then we may extend $V0$ to an invariant V for singular knots as follows.

Suppose that V has already been defined for singular knots with at the most n-1 vertices.

Now, let K be a singular knot with n vertices, and further K,+,$K-$ are singular knots that are an equivalent everywhere except ata neighbourhood of onevertex.

In this neighbourhood they differ only within the way shown in figure

K $K_{+}K_{-}$

Then we all define,

$$\mathbf{V(K)} = \boldsymbol{V}(\boldsymbol{K}_{+}) - \boldsymbol{V}(\boldsymbol{K}_{-})$$

Since K has n vertices and both K_{+} *and* K_{-} have n-1 vertices, we may by hypothesis evaluate V(K).

Preposition:

For any integer l ≥0,

1. $\nabla_{(n;0)}$(z) = $\nabla_{k(n;l)}$(z)
2. $\nabla_{(n;0)}$(z) = 1-2z^n+ ⋯ if n iseven.
3. $\nabla_{(n;0)}(z)$ = 1-2z^{n+1}+ ⋯ if n isodd.
4. $V_{(n;l)}$(t) = $t^{2l}(V_{k(n;0)}(t) - 1)$+1

Preposition:

Let V be a vassiliev invariant or order O. Then for any non- singular knot K,

v(k)= v(O)

Therefore there's essentially just one vassiliev invariant of order zero.

Proof:

Since v is of order 0, it follows that V(⤧)=0, and hence from the singular skein relation,

V (⤱) = V (⤲)

This implies that if we apply an unknotting operation at any crossing point of k, then the worth of v remains constant.
Since a knot are often deformed may be a trivial knot by applying several unknotting operations.
It follows that v(k) = v(O), therefore v may be a constant for any non- singular

knot.

Preposition:

Let $\emptyset(t)$ be the Taylor expansion of the Jones polynomial of a knot k at t=1, which we may write as

$$\emptyset(t)= C_0 + C_1(t-1) + C_2(t-1)^2 + \cdots + c_m(t-1)^m + \cdots$$

Where if we recall some first-year calculus

$$C_m = \frac{1}{m!}\left[\frac{d^m v(t)}{dt^m}\right]_{t=1}$$

Then,

$$\boldsymbol{\emptyset(t) = C_m}$$

Is a Vassiliev invariant of order (at most) m.

Preposition:

Two singular knots K and K' are equivalent if and only if there exist singular diagrams D and D'. Respectively which can be deformed into each other by applying a finite number of times

1) The Reidemeister moves or their inverses except within a small neighbourhood of eachvertex.
2) The following operation $\bar{\Omega}$, near thevertices.

Preposition:

None of the following classical geometric invariants:

The minimal crossing number c(k), the unknotting number u(k), the bridge number b_r(k), the braid index b(k) and the genus g(k) of a knot, k is of a finite type;

Hence they cannot be Vassiliev invariants.

Theorem:

For a knot K and n≥2 , let $K' = K(n; l)$, $l \geq 0$, be the connected sum of K and $K(n: l)$ then

i) $K'$is not equivalent toK.

ii) For any vassiliev invariant $V_m$of order m, if $m \leq n$then

$$V_{m}(K') = V_{m}(K)$$

Proof :

Therefore there are infinitely many distinct knots that can't be distinguished by finitely many vassiliev invariants.

In the above example $K'$is not a major knot.

However there does exist an example of an equivalent property as above during which both K and K' are prime knots.

If on the opposite hand we consider "all"vassiliev invariants then things is sort of different.

One of the strengths of the idea that surrounds at the vassiliev invariants is that it allows us to treat the polynomial invariants during a systamatic way.

Hence the vassiliev invariants may reveal relationships between the polynomial invariants.

In the particular case a surprising results of this type has been found.

Preposition:

The singular knot invariant V defined by the signature of a knot K is not of finite type; hence it is not a vassiliev invariant.

Proof:

Let us assume that V is of order at the most (≥ 2).

We shall show that $(k[n + 1, n]) \neq 0$ or equivalently,

$\sum(-1)l\ \sigma(k[n + 1, n]\varepsilon 1,\varepsilon 2,\ldots..\varepsilon n{+}1) \neq 0$

Now,

since $k[n + 1, n]\varepsilon 1,\varepsilon 2,\ldots..\varepsilon n{+}1$ may be a torus knot type $(2n - 2l + 1,2)$its signature.

i) If $l \neq n\ ,n + 1,\ \sigma(k[n + 1, n]\varepsilon 1,\varepsilon 2,\ldots..\varepsilon n{+}1) = -(2n - 2l)$

ii) If $l = n\ or\ n + 1,\ \sigma(k[n + 1, n]\varepsilon 1,\varepsilon 2,\ldots..\varepsilon n{+}1) =0$

It is quite easy to ascertain that the amount of knots with l negative signs within the sequence $\varepsilon 1,\ \varepsilon 2,\ \ldots\ .\ .\ \varepsilon n{+}1\ is\ (n{+}1)$.

So the left side is

$\sum(-1)l\ \sigma(k[n + 1, n]\varepsilon 1,\varepsilon 2,\ldots..\varepsilon n{+}1) = \sum n{-}1(-1)l{+}1\ (n{+}1)\ (2n - 2l)$

$l{=}0\ l$

Which can be shown to be non-zero.

This now contradicts the idea that V is that if order at the most n.

Preposition:

The n^{th}co-efficient of the polynomial ∇ is a vassiliev invariant of degree n.

Proof:

The skein relation shows that $\nabla(l)$ is divisible by t^nif L is a singular link with at least n double points and the n^{th}co-efficient a_n of the polynomial ∇ is an integral invariant of oriented links which vanishes on every singular links with at least $n + 1$ doublepoints

The result follows

If E is a module, denote by I(E) the set of invariants of knots with valuesinE.Foreveryinteger$n \geq 0$,denoteby$V_n(E)$thesetofvassiliev invariants of degree ≤ 0.

Preposition

LetRbearing.ThentheRmodulus$V(R)$fromanincreasing family of finitely generated R-submodules of I(R).

$$V_0(R) \subset V_1(R) \subset V_2(R) \subset \cdots \subset I(R)$$

Moreover one has:

$\boldsymbol{VP(R)V_q(R) \subset V_{p+q}\ for\ every\ p, q \geq 0.}$

CONCLUSION

I have made a survey of what has led to the study of knots in mathematics, the relevance of the study, the fundamental concepts which are dealt with in the theory, the slow development of the subject and its application in the inter disciplinary field.

This is a brief report of a small area of the vast field of Knot Theory. The knots can be studied classifying them as torus knots, satellite knots and hyperbolic knots. Every knot or link is proved to be a closed braid.

Knots are being approached using the theory of braids as well. The concept of fundamental groups has helped in defining the Knot group. Thus Knot Theory can be studied connecting it with abstract algebra. A signed graph can be drawn corresponding to a link and vice versa.

This provides a bridge between knot theory and graph theory. The study of Reidemeister moves, some classical invariants like crossing number, knotting number, bridge number and other invariants like the genus of a knot and some polynomial invariants have been discussed here.

The survey has shown that the fundamental problem of knot theory was the process of distinguishing knots. Many invariants have been discovered to show that two knots are not equivalent. If an invariant of two knots is equal it did not necessarily imply that the knots are equivalent.

This necessitated further research o Also classifying knots and studying them with a topological point of view is becoming essential in the inter disciplinary field as well leading to a great scope to explore thefield.

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