A STUDY ON THE JONES REVOLUTION

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ABSTRACT

In this section we introduce the Jones polynomial a link invariant found by Vaughan Jones in 1984.

Later on, Louis Kauffman expanded further Jones study of knot polynomials, allowing for a much simpler presentation of the Jones polynomial.

INTRODUCTION

In this chapter our intension is to study the new invariants from the point of view of knot theory, explaining several of their fundamental properties.Also in the final section we shall show an application of the Jones polynomial to knot theory itself,namely solving a couple of the Tait conjectures the original knot theory conjectures.

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The Jones polynomial:

Let us by defining the Jones polynomial from the perspective of knot theory, rather than say operator algebras or quantum groups.

Which would require the introduction of a great deal of new notation and definitions without significantly illuminating our further discussions.

The Jones polynomial is an X polynomial obtained by replacing A by $t^{-1/4}$.

Jones polynomial

- Step 1 to step 3 are all same as Alexander
  Jones skein relation is,

$$t^{-1}V_{L_+} - tV_{L_-} = (t^{1/2} - t^{-1/2})V_{L_0}$$

Where V_{L_+}, V_{L_+} and V_{L_0} are

L_+ :  L_- :  L_0 : 

- And of course, the Jones Polynomial of the Unknot is 1!

Theorem:5.1

Suppose $-K$ is the knot with the reverse orientation to that on K,

then,

$$\boldsymbol{V_{-k}(t) = V_{k}(t)}$$

Proof:

In order to calculate $V_{-k}(t)$ we may use the same Skein tree diagram as for the calculation of the $V_{k}\ (t)$.
Hence the result follows,

As a consequence of the above theorem the Jones polynomial is not a useful tool in the study of whether or not a knot is invaertible.

However the Jones polynomial is a powerful tool in the study of amphicheirality of a knot.

Theorem:5.2

Suppose that K is a (oriented) knot (or link) then,

1) $V_k(t) = P_k\left(t, \sqrt{t} - \frac{1}{\sqrt{t}}\right)$

2) $\Delta_k(t) = P_k\left(1, \sqrt{t} - \frac{1}{\sqrt{t}}\right)$

Proof :

So ,it may be said that $V_k(t)$ and $\Delta_k(t)$ are special cases of $P_k(V, Z)$.

As we have already mentioned they are essentially polynomials.

To calculate $S_k(x. y, w)$ and $P_k(v, z)$ for an arbitrary knot (or link) it is better to use the Skein diagram.

Exercise:5.7

1) Show that if K is a knot then $\Delta_k(z)$ is an integer polynomialin z^2.
2) Show that if K is a μ −component link, then we maywrite $\Delta_k(z) = z^{\mu-1}g(z)$ is an integer polynomial in z^2.

Proof:

In this section we have described an extremely efficacious method to calculate the Alexander polynomial for an arbitrary knot (or link).

In fact, this method has been taken up with great gusto since it was first introduced by Conway, and innumerable Alexander polynomials have been calculated.

This method is still very powerful if in our research we want to calculate the Alexander polynomial of a specific knot (or link).

By the 1960s, however, the Alexander polynomial had already been used to the point of exhaustion to detect Global properties of knots (or links).

That is to say, it would seem to be a futile exercise to 0 just carryon calculating the Alexander polynomial for knots (or links) with arbitrary large number of crossing points, since it is very possible that no further insight into the Global properties of knot (or link) will be garnered by so doing.

Around 15 years after Conway introduced his method, it was shown that in addition to the Alexander polynomial, his approach is very useful in the calculation of the "new" knot invariants.

CONCLUSION

I have made a survey of what has led to the study of knots in mathematics, the relevance of the study, the fundamental concepts which are dealt with in the theory, the slow development of the subject and its application in the inter disciplinary field.

This is a brief report of a small area of the vast field of Knot Theory. The knots can be studied classifying them as torus knots, satellite knots and hyperbolic knots. Every knot or link is proved to be a closed braid.

Knots are being approached using the theory of braids as well. The concept of fundamental groups has helped in defining the Knot group. Thus Knot Theory can be studied connecting it with abstract algebra. A signed graph can be drawn corresponding to a link and vice versa.

This provides a bridge between knot theory and graph theory. The study of Reidemeister moves, some classical invariants like crossing number, knotting number, bridge number and other invariants like the genus of a knot and some polynomial invariants have been discussed here.

The survey has shown that the fundamental problem of knot theory was the process of distinguishing knots. Many invariants have been discovered to show that two knots are not equivalent. If an invariant of two knots is equal it did not necessarily imply that the knots are equivalent.

This necessitated further research o Also classifying knots and studying them with a topological point of view is becoming essential in the inter disciplinary field as well leading to a great scope to explore thefield.

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