# A STUDY ON PARTIAL DIFFERENTIAL EQUATION USING MONGE'S METHOD

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**Abstract:**

In this paper it discuss about the partial differential equation of order two with variable coefficients. Here it explains how a large class of **partial differential equations using Monge's method.** and the Monge's method for solving some important type of second order partial differential equation.

**Keywords:**

Monge's method, intermediate integral, complete integral.

## 1. Introduction:

A partial differential equation is said to be of order two , if it involves least one of the differential coefficients r,s,t . the general form of a second order Partial differential equation is two independent variables x, y, is given as

$$p = \frac{\partial z}{\partial x}, \; q = \frac{\partial z}{\partial y}, \; r = \frac{\partial^2 z}{\partial x^2}, \; s = \frac{\partial^2 z}{\partial x \partial y}, \; t = \frac{\partial^2 z}{\partial y^2}$$

the most general linear partial differential equation of second order in two independent variable x and y with variable coefficient is given as

$$Rr + Ss + Tt = V$$

## 2. Preliminaries:

**Definition:**

An **intermediate integral** of a partial differential equation system E is another PDE system of lower order , whose solution are also solution of E the derivation of this relies only one well known properties of first order PDE ,systems and some elementary linear algebra.

**Definition:** A function is any algorithm or relation that relates a bunch of input equalities to a bunch of output quantities such that each input corresponds to one and only one output.

**Definition:**

**Complete integral**

A solution of a partial differential equation of the second order that contains as many arbitrary constants as there are independent variables.

**3. Monge's method of integrating $Rr+Ss+Tt+U(rt-s^2)=V$ …(1)**

Where r,s,t have their usual meanings and R,S,T,U,V are Functions of x,y,z,p and q.

We have $dp = rdx + sdy$, $dq = sdx + tdy$

$$\therefore \quad r = \frac{dp - sdy}{dx} \quad \text{and } t = \frac{dq - sdx}{dy}$$

$$R\,\frac{dp - sdy}{dx}\,Ss + T\,\frac{dq - sdx}{dy} + U\left\{\frac{(dp - sdy)(dq - sdx)}{dxdy} - s^2\right\} = V \qquad \text{(or)}$$

$$(Rdpdy+Tdqdx+Udpdq-Vdxdy)-s(Rdy^2 - S\,dxdy+Tdx^2+Udpdx+Udqdy) = 0$$

Monge's subsidiary equations are

$$L=Rdpdy+Tdqdx+U\,dpdq-Vdpdy\,) = 0 \qquad \textbf{… (2)}$$

And $$M=Rdy^2-Sdxdy+Tdx^2+U\,dpdx+Udqdy \qquad \textbf{… (3)}$$

Here the equation (3) cannot be resolved into two linear equationson on account of the presence of the term

$Udpdx+Udqdy$ .

However we try to resolve M+λ L = 0 into two linear equations , where λ some multiple to be determined .

Now

$M+\lambda L = Rdy^2+Tdx^2(S+\lambda V)dxdy+Udpdx$

$$+Udqdy+\lambda Rdpdy+\lambda Tdqdx+\lambda Udpdq \quad \textbf{....... (4)}$$

Also le tM+λL = ( αdy +βdx+γdp) ( α'dy + β'dx+γ dq ) **...( 5)**

Equating the coefficients of $dy^2$,$dx^2$ and dpdq in ( 4) and (5) ,

We have

$$R= \alpha\alpha', \quad k = \beta\beta', \quad \lambda U \quad \gamma\gamma'.$$

If we choose $\alpha' = 1, \beta' = \frac{1}{k'}, \ \gamma' = \frac{\lambda}{m}$ then

$$\alpha = R, \quad k = kT, \ \gamma = mU$$

Now equating the coefficients of the remaining five terms in (4) and (5) , we have

$$kT+\frac{R}{k}=-(S+\lambda V) \quad \textbf{...( 6)}$$

$$\frac{kT\lambda}{m} = \lambda T , U = \frac{mU}{k} \quad \textbf{...( 7)}$$

$$\frac{\lambda R}{m} = U , \lambda R = mU \quad \textbf{...( 8)}$$

From (7), m =k and from (8), $m = \frac{\lambda R}{U}$

Putting, $k = \frac{\lambda R}{U}$ in (6), we have

$$\frac{\lambda R}{U} T + R \frac{U}{\lambda R} = - ( S +\lambda V)$$

$$\lambda^2 (RT+UV ) +\lambda US + U^2 =0 \quad \textbf{...( 9)}$$

Apart from the special case when $S^2 = 4$ ( RT+ UV) , this equation will have two distinct roots $\lambda_1 , \lambda_2$ .

for $\lambda = \lambda_1$ , $m = k = \frac{R\lambda_1}{U}$ . then from (5) , M+ λL = 0 gives

$$(Rdy+\lambda_1 \frac{RT}{U} dx+\lambda_1 Rdp) (dy+\frac{U}{R\lambda_1}dx+ \frac{U}{R} dq) = 0 \qquad \text{(or)}$$

$$(Udy+T \lambda_1dx+U\lambda_1dp) (R \lambda_1dy+Udx+U \lambda_1dq)= 0 \qquad \textbf{...( 10)}$$

Similarly for $\lambda = \lambda_2$ , $M = \lambda L = 0$ gives

$$(Udy+T\lambda_2dx+U \lambda_2dp) (R\lambda_2dy+Udx+U\lambda_2dq) = 0 \qquad \textbf{...( 11)}$$

Now one factor of (10) is combined with one factor of the first factor of (10) with the first factor of (11) or the second factor of ( 10 ) with the second factor of (11) .

$$Udy+\lambda_1Tdx + \lambda_1Udp = 0 \} \qquad \textbf{...( 11)}$$

$$\lambda_2Rdy+Udx+ \lambda_2Udq = 0 \qquad \textbf{...( 12)}$$

and

$$Udy+\lambda_2Tdx+\lambda_2 Udp = 0\} \qquad \textbf{...( 13)}$$

$$\lambda_1Rdy+Udx+\lambda_1Ud=0 \qquad \textbf{...( 14)}$$

From each of these pairs we shall derive two integrals of the form $u=a$ , $v=b$. Let $u_1=a_1$,

$v_1 = b_1$ be the integrals obtained from the equations ( 12) $u_2=a_2$ , $v_2=b_2$.Be the integrals obtained from the equations (13) . then the two intermediate integrals are

$$u_1=f_1 (v_1) \text{ and } u_2 = f_2(v_2)$$

Which can often be solved to find the values of p and q as functions of x,y and z substituting these values of p anq in $dz = pdx + qdy$ and integrating it we obtain the solution of the original equation

**PROBLEM:**

**(1)Solve:3r+4s+t+(rt-s$^2$)=1 By Using Monge's Method** $\rightarrow$ (1)

**Solution:**

Comparing the given equation (1) with

$$\mathbf{Rr + Ss + Tt + U(rt - s^2) = V}$$

We have

R=3, S=4, T=1, U=1, V=1

The $\lambda$ -quadratic equation is

$$\lambda^2 \mathbf{(UV+RT)} + \lambda \mathbf{SU+U^2=0}$$

$$\lambda^2 (1(1) + 3(1)) + \lambda (4(1)) + 1^2=0$$

$$4\lambda^2+4\lambda+1=0$$

$$4\lambda^2+2(2)\lambda(1)+1=0$$

$$(2\lambda+1)^2$$

$$(2\lambda_1+1)(2\lambda_2+1)$$

$$2\lambda_1=-1, \quad 2\lambda_2=-1$$

$$\lambda_1=-\frac{1}{2}, \quad \lambda_2=-\frac{1}{2}$$

In this case we can find only one intermediate integral, which is given by the equation

$$\mathbf{Udy+\lambda_1 Tdx+\lambda_1 Udp=0}$$

$$1dy - \frac{1}{2}(1)\,dx - \frac{1}{2}(1)dp = 0$$

$$dy - \frac{1}{2}\,dx - \frac{1}{2}\,dp = 0$$

$$2dy-dx-dp=0$$

$\lambda_2$ **Rdy + Udx +** $\lambda_2$**Udq =0**

$-\frac{1}{2}(3)dy + (1)\,dx - \frac{1}{2}(1)\,dq = 0$

$-\ \frac{3}{2}\,dy + dx - \frac{1}{2}dq = 0$

$3y - 2x - q = b \qquad \rightarrow (2)$

The intermediate Integral is

$-2y+x+p=f(3y-2x+q) \qquad \rightarrow (3)$

From (2), p=2y-x+a , q=-3y+2x+b

Putting these values of p and q in dz=pdx+qdy

We get

$dz=(2y-x+a)dx+(3y+2x+b)dy$

$=2ydx-xdx+adx+3ydy+2xdy+bdy$

$=2xy-\frac{1}{2}x^2-\frac{3}{2}y^2+ax+by+c$

**4. CONCLUSION:**

In this paper presentation briefly discussed partial differential equation using Monge’s method .also we have discussed some basic definition and some examples and we have discussed about solution of the of the Monge’s method of integrating Rr +Ss+Tt = V.

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