A STUDY ON ELECTRO STATICS

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ABSTRACT:

In this paper we are elaborate Electrostatics is based primarily inverse law. Then small volume distance between two forces. Also we solve the Uniqueness of the solution of an electrostatic problem .

Keywords: Electro , vector , volume , surface , direction ,magnitude

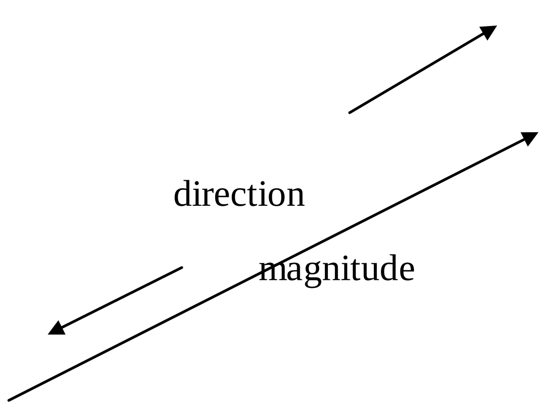
Introduction: Vector calculus, or vector analysis is a branch of mathematics concerned with differentiation and integration of vector fields.

"Vector calculus" is some times used as a synonym for the broader subject of multivariable calculus, which includes vector calculus as well as partial differentiation and multiple integration. Electrostatics is concerned with electricity at rest. This electricity may be in the form of discrete point charges or in the form of a continuous charge distribution with volume density ρ or surface density ς or even in the form of electric dipoles, where a dipole may be regarded as the limiting case of two large equal and opposite point charges at a small distance apart. Volume and surface distribution of dipoles also play an important part in the theory. The study of Electrostatics is based primarily on columb"s inverse square law.

BASIC DEFINITIONS:

DEFINITION : 1.1

A quantity which has both magnitude as well as direction is called vector.



DEFINITION: 1.2

- A scalar is a physical quantity with magnitude only.
- A vector is a physical quantity with magnitude and direction.
- A unit vector has magnitude one.
- The position vector r = (x, y, z)

DEFINITION : 1.3

It is branch of mechanics which deals with the motion of bodies under the action of given forces.

DEFINITION : 1.4

The term 'particle' we mean that a finite mass occupying a point in the Eulidean space. In fact this is a purely mathematical concept.

In physics by the term 'particle' we mean a mass occupying an infinitesimally small amount of volume approximately compared to the distance between masses.

Example :

The earth and the moon may be thought of as particles approximately, compared to their distance apart.

DEFINITION : 1.5

When a constant force F acts upon a particle for a time – interval Δt then the product $F\Delta t$ is called the impulse.

Then $F = \frac{dp}{dt}$

If the force acts from a time t_1 to a time t_2 then the change in momentum during this time – interval is

$$P_2 - P_1 = \int_{P_1}^{P_2} dp = \int_{t_2}^{t_1} F.dt$$

DEFINITION : 1.6

Work is said to be done by a force when its point of application undergoes a displacement.

DEFINITION : 1.7

In physics, motion is the change in the position of an object over time.

Motion is mathematically described in terms of displacement, distance, velocity, acceleration, speed, and time.

DEFINITION : 1.8

A measure of how much matter is in an object mass is measured in grams and pounds.

DEFINITION: 1.9

$$W = \underline{F}.\underline{AB}$$

[work done(w) by the force $\underline{F}$ during the displacement $\underline{AB}$ the scalar quantity]

DEFINITION : 1.10

The rate change of displacement with respect to time is known as velocity. It is a vector quantity because is has both magnitude and direction velocity is denoted by $\underline{v}$.

The Inverse Square Law : 2.1

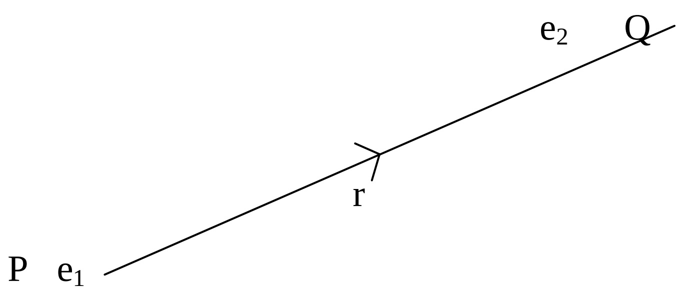
If two electric charges e_1, e_2 are placed in vacuo at a distance r apart, the force between them is proportional to

$$\frac{e_1 e_2}{r^2}$$

Further if the charges are of the same sign, the force is repulsive if they are of opposite signs, the force as attractive.

Thus the force F on a charge e_2 at Q due to another charge e_1 at P is given by

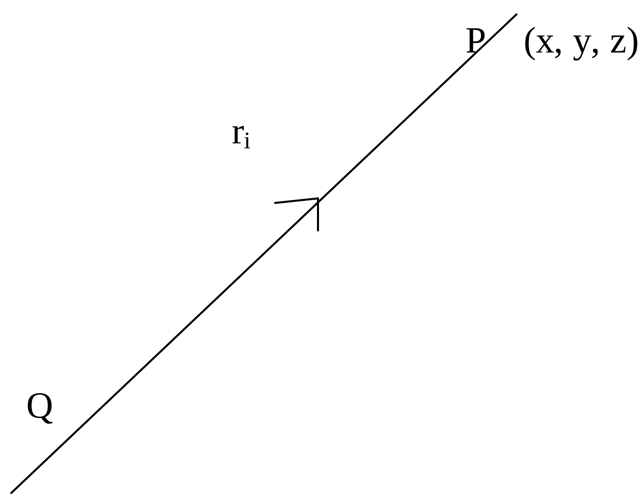
$$F = C\ \frac{e_1 e_2}{r^2}\frac{r}{r} = c\frac{e_1 e_2}{r^3}r$$



Where C is a constant and r is the position – vector of a relative to P. In C.G.S electrostatic unit system, we define a unit charge as one which, in vecuo, repels an exactly similar charge at a distance of one cm with a force of one dyne. In this system c = 1 and

$$F = \frac{e_1 e_2}{r^3}r \text{ ....(1)}$$

This gives both magnitude and direction of the force, for if e_1, e_2 ar of like signs, the force on Q is refulsive, if of opposite signs, the force is attractive.



We define the electrostatic intensity or the electrostatic field vector at a point as the force that would act in vacuo on a unit charge placed at that point, due to the rest of the charges, provided the unit test charge is assumed not to disturb the field due to the other charges. This intensity has at each point a definite magnitude and a definite direction and is thus a vector. We denoted it by E. Thus for n charges

$e_1, e_2, \ldots\ldots\ldots, e_n$

$$E = \sum_{i=1}^{n} \frac{e_i}{r_i^3} r_i \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Where r_i denotes the position vector of P relative to the i^{th} charge e_i.

Now

$$r_i^2 = (x - \varepsilon_i)^2 + (y - v_i)^2 + (z - \zeta_i)^2$$

$$\nabla\left(\frac{1}{r_i}\right) = -\frac{1}{r_i^2}\left(\frac{x - \varepsilon_i}{r_i} i + \frac{y - \eta_i}{r_i} j + \frac{z - \zeta_i}{r_i} k\right)$$

$$= -\frac{1}{r_i^3} r_i$$

If we define

$$\emptyset = \sum_{i=1}^{n} \frac{e_i}{r_i} \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Then

$$\nabla\emptyset = \sum_{i=1}^{n} \nabla\left(\frac{e_i}{r_i}\right) = -\sum_{i=1}^{n} \frac{e_i}{r_i^3} r_i = -E$$

$$E = -\nabla\emptyset = -grad\emptyset \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Thus further from (2)

$$divE = \nabla\left[\sum_{i=1}^{n} \frac{e_i}{r_i^3} r_i\right]$$

$$= \sum_{i=1}^{n} e_i \nabla \left(\frac{r_i}{r_i^3}\right) = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

E is a solenoidal vector.

Also $curl\, E = -\Delta\ x\ (\Delta\emptyset) = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(6)$

E is also an irrotational vector.

Combining (4) and (5) we have

$$\nabla(\nabla\emptyset) = 0$$

Or

$$\nabla^2\emptyset = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots(7)$$

Therefore at a point not occupied by charges $\emptyset$ satisfied Laplace's equation (7)

We can verify (6) in an alternative manner. For any closed path, that does not include a charge, we have

$$\oint E.dr = -\oint \nabla\emptyset\, dr = -\oint\ d\emptyset = 0$$

$$curl\, E = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(8)$$

But $\oint\ E.dr$

Represent the work done by the electric intensity, when a unit positive charge goes round a closed path. Thus electrostatic intensity is a conservative force and the work done in carrying a unit positive charge from one position to another does not depend on the path, but only on the position – vector of the end point.

Now

$$\int_P^\infty\ E.dr = -\int_P^\infty\ d\emptyset = \emptyset(P) - \emptyset(\infty) = \emptyset(P)$$

As from (3)

$$\emptyset(\infty) = 0$$

Thus $\emptyset$ can now be interpreted as the work done by the electrostatic intensity in carrying unit positive charge from P to infinity along any path what so ever or as the work done against the electrostatic force in bringing the unit positive charge from infinity to P.

Some Fundamental Definitions :

DEFINITION : 2.1

The scalar point function $\emptyset$ defined in $\emptyset = \sum_{i=1}^{n} \frac{e_i}{r_i}$ is called the electrostatic potential of the field due to the given system of charges. The sufaces

$$\emptyset(x, y, z) = constant$$

For various values of the constant constitute a family of equipotantial surfaces. The vector equation of the family is

$$d\emptyset = dr.\nabla\emptyset = 0$$

$$dr.E = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots..(1)$$

Which shows that no work is done in going from one point to another along an equipotential surface.

DEFINITION : 2.2

A line of force is a curve such that the tangent to it at any point gives the direction of E at the point.

The vector differential equation of the lines of force is

$$\text{dr x E} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Through every point in space in general, only one line of force will pass, but when E = 0 the direction of the line of force becomes indeterminate such a point is called a nalltral point or a point of equilibrium.

DEFINITION : 2.3

The flux of a vertor E across the surface element da is defined as the scalar E. da If S is any surface, closed σr open the flux of E across S is

$$\int E.da$$

S

If the surface is closed the flux of E out of S is

$$\int_{s} E.da$$

Where the direction of da is that of the outward drawn normal.

The flux of E out of the surface of a sphere of radius a, at the centre of which is placed a charges e, is

$$\int_{s} E.da = \int_{s} \frac{e}{a^2} 4\pi a^2 = 4\pi e$$

Gauss's Theorem :2.2

Statement :

If N is the flux of the electrostatic intensity E out of any closed surface S , then

$$N = \int_{s} E.da = 4\pi Q \quad \text{……………………(1)}$$

Where Q denotes the total charge enclosed by the surface S.

Proof :

$$\int_s E.da = \int_s \left[\sum_{i=1}^{n} \frac{e_i r_i}{r_i^3}\right] da$$

$$= \sum_{i=1}^{n} e_i \int \frac{r_i}{r_i^3}\, da$$

$$= \sum_{i=1}^{n} e_i \int d\omega_i = \sum_{i=1}^{n} e_i \omega_i$$

Where ω_i is the solid angle subtended by the closed surface at the i^{th} charge. Now from,

(i) At an internal point. The whole area of the unit sphere = 4π

(ii) At an point on the surface : surface area of half the unit sphere = 2π

(iii) At an external point : solid angle subtended is zero.

And $\omega_i = 4\pi$ (or) 0 according as e_1 is inside or outside the surface.

Hence the theorem follows.

Note :1

For a point charge, if any, on the surface S itself, $w_i = 2\pi$ and in this case, the theorem becomes

$$N = 4\pi Q + 2\pi Q' \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Where Q' denotes the total charge on the surface.

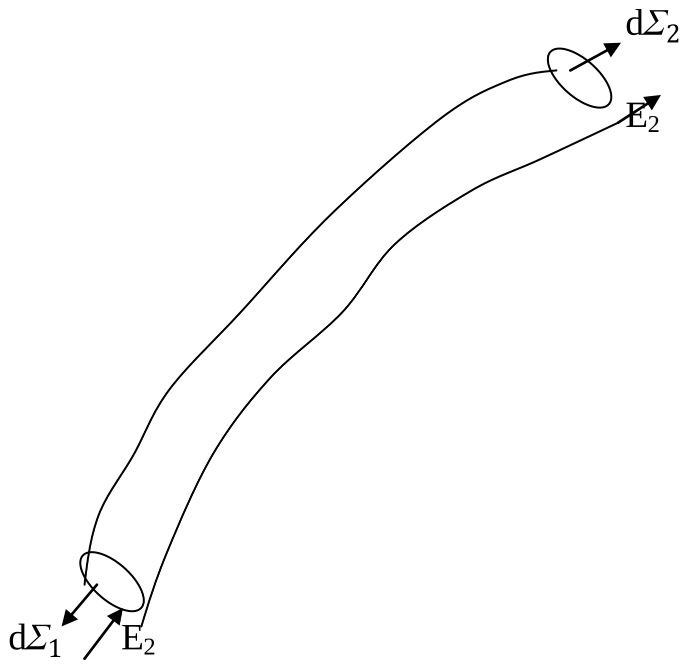
Note :2

We have proved Gauss's theorem on the hypothesis that all the charges are discrete. But if the charge distribution is continuous the contribution to N by an element dv of charge is $4\pi\rho dv$ or zero according as the charge is inside or outside

S. Integration through all the space occupied by the charge distribution proves the theorem.

Theorem: 2.3

The strength of a tube of force (or) the flux across any section of it) remains constant along its path.



As there is no flux out of a tube of force, applying Gauss's theorem to the closed surface show in the figure.

$$E_1 \,.\, da + E_2 \,.\, da_2 = 0$$

$E_1\, ds_1 = E_2\, ds_2$

Uniqueness of the solution of an electrostatic problem : 2.4

Problem :2.4.1

Let $\emptyset_1$ and $\emptyset_2$ be the two possible potential distributions due to given charge distribution ρ and surface distribution σ on the surface of given conductors. Then in space, we have

$$\nabla^2\emptyset_1 = \nabla^2\emptyset_2 = -4\pi\rho$$

and on the surface of conductors, we have

$$\nabla\emptyset_1 = \nabla\emptyset_2 = -4\pi\sigma_n$$

and that if $\theta = \emptyset_1 - \emptyset_2$

Then $\nabla^2\theta = 0$ in all space

and $\nabla\theta = 0$ on the surface of all conductors. Now from divergence theorem

Now from Divergence theorem

$$\int (\theta\nabla\theta\ ).da = \int div(\theta\nabla\theta).dv$$

$$= \int \theta\nabla^2\theta + (\nabla\theta)^2\, dv \ldots\ldots\ldots\ldots\ldots\ldots..(1)$$

But $\nabla^2\theta = 0$ in all space and $\nabla\theta = 0$ on the surface of the conductors.

Also from considerations of magnitude the surface integral vanishes over the sphere at infinity.

$$/ \int_v (\nabla\theta)^2 dv = 0$$

Which implies that $\nabla\theta = \theta$ in all space.

or $\theta = constant.$

Or $\emptyset_1 - \emptyset_2 = constant$

But $\emptyset_1 = \emptyset_2$ over the surfaces of conductors

$\emptyset_1 = \emptyset_2$ in all space.

Thus the potential distribution must be unique.

Coulomb's Law : 2.5

Consider a charge q at rest with position vector $\vec{r_1}$. This is called the 'source' charge. When a test charge Q with position vector $\vec{r}$ is placed at a distance $\vec{r}$ away, the force is given by coulomb's Law.

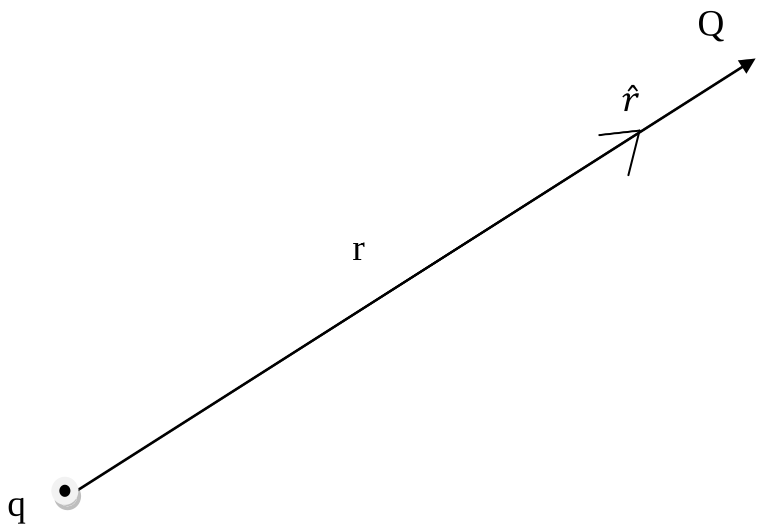


Fig (a)

Coulomb showed experimentally that a force on the point charge Q due to the single point charge q at rest is

1) Proportional to the product of (Their magnitudes) of the charge.
2) Inversely proportional to the square of the distance separating them ($\vec{r}$). Fig (a)
3) Acts along the line joining the two charges

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r^2}.\hat{r} \quad \ldots\ldots\ldots\ldots\ldots..(1)$$

$\in_0$ is called the permittivity of free space

$$= 8.85\ x\ 10^{-12}\ \frac{c^2}{N.m^2}$$

$\hat{r}$ = unit vector in the direction of $\vec{r}$

$\vec{r}$ is the separation vector from $\vec{r}(q) to\ \vec{r}$ (test charge Q)

i.e. $\vec{r} = \vec{r} - \vec{r}$ r is its magnitude.

The force is directed from q to Q. If the force is repulsive the charges have same sign and is of oppositive sign if the force is attractive.

Principle of Superposition : 2.6

The principle states that the interaction between any two charge is completely unaffected by the presence of other charges. To determine the force on Q, we find the force F_1 due to q_1 alone (ignoring all the other charges) then we find the force F_2 due to q_2 alone and so on.

The total force $\vec{F}$ is the vector sum of the individual forces.

$$\vec{F} = \overrightarrow{F_1} + \overrightarrow{F_2} + \overrightarrow{F_3} + \cdots \ldots .\overrightarrow{F_n}$$

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 Q}{r_1^2}\widehat{r_1} + \frac{q_2 Q}{r_2^2}\widehat{r_2} + \cdots \ldots \ldots .\frac{q_n Q}{r_n^2}\widehat{r_n}\right]$$

$\overrightarrow{r_1}, \overrightarrow{r_2} \ldots \ldots .. \overrightarrow{r_n}$ are distances of charges $q_1, q_2 \ldots \ldots \ldots \ldots \ldots . q_n$ from and $\widehat{r_1}, \widehat{r_2} \ldots \ldots \ldots \ldots . \widehat{r_n}$ are unit vectors in the direction of $\widehat{r_1}, \widehat{r_2} \ldots \ldots \ldots \ldots . \widehat{r_n}$ respectively.

The Electric Field : 2.7

If we place a charge Q at a distance $\vec{r}$ from a charge q, it experiences a force. Hence the region surrounding the charge q in which it exerts an electric force on another charge Q is termed as the electric field of charge q.

The intensity of the electric field at a given point is defined as the electric force experienced by a unit positive charge placed at that point.

If a positive charge q experiences a force $\vec{F}$, then the electric field intensity is

$$\vec{E} = \frac{\vec{F}}{q}$$

Or in general

$$\vec{E} = \frac{\vec{F}}{q} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

The electric field intensity $\vec{E}$ at a point is defined as the limiting value of the ratio of the force $\vec{F}$ experienced by a charge q placed at that point, as the value of q tends to zero.

$$\vec{E} = \frac{q}{4\pi\varepsilon_0 r^2}\hat{r} \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

For several point charges $q_1, q_2 \ldots\ldots\ldots\ldots\ldots q_n$

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n} \frac{q_i}{r_i^2}\widehat{r_i} \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

Concept of a field :2.8

Fields are not composed of matter (electrons, protons and neutrons etc), but they be still exist. Fields can be considered as a complementary component of the world to the particles of matter. Particles are localised with a relatively well – defined position whereas fields extend throughout all of space.

‘Fields are found to affect matter, and matter to affect fields, and it is through their mutual effects that we determine the properties of both matter and fields. For example, consider two particles X and Y. we can imagine the interaction between x and y as a three stage process :

Particle x affects the field around it.

The disturbance of the field around x spreads away from x.

A little later, particle y is affected by the disturbance of the field created by x.

Notice that the field itself is not an observable ‘object’ we can infer its properties from systematic studies of the way one particle affects other particles around it. Currently we can explain all observable phenomena in the universe in term of three different types of fields. The gravitational, electro weak and strong fields. It is hoped that all these fields ÷ will be understood on the basis of a ‘unified’ field. In coulomb’s law, the force exerted by one particle on another is seen as an action at a distance without the in between space. But there is a finite time required for electromagnetic interactions to be propagated. This time is the time for the speed of light to travel in the given medium. Thus the force is not transmitted in stantaneously with infinite speed but requires some time.

CONCLUSION :

The study of this project is vector Analysis in applications of vector calculus. I have considered, work, D’Alemberts principle and General Equation of motion of a Rigid body of vector calculus. Wheel related calculus shell related algorithm, theorem, problem and some operations of applications of vector calculus. Though this work deal with only basic ideas and concept may be extended in the application of some field.

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