# AN STUDY ON FUZZY COLORING APPROACH

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***ABSTRACT***

*The field of Maths assumes an indispensable job in different fields.Graph theory is asignificant zonein Maths, utilized for several models. An old style graph speaks to an old style connection among objects. The items are spoken to by vertices & relations by edges. Coloring of Graph is a considered issue of combinatorial streamlining. Theory of F-graphs has various applications in current science & innovation, particularly in neural networksfields, information theory, cluster analysis, expert systems, control theory, medical diagnosis, etc. In 1975Rosenfeldpresented the concept of F-graphs. The coloring of F-graphshas a few applications in reality. Graph coloring fills in as a model for compromise in issues of combinatorial improvement. F-graphsColoring has some genuine combinatorial streamlining applications issues as exam schedule, allocation registration, systemof traffic light, and so forth. The most significant issue in the coloring issue of the F-graphis to develop a strategy for finding F-graphschromatic numbers. In this Article, we analyzed Coloring Approach in Fuzzy Graph Theory.*

## I. INTRODUCTION

These days, some true issues can't be appropriately demonstrated by fresh graph theory, since the issues contain dubious information. The cause oftheory of graph began in 1735with the issueof connecting by Konigsberg. The fuzzy graph is increasingly adaptable, exact, and perfect than the old-style graph to display ensure genuine issues. The fuzzy graph approach is more remarkable in cluster analysis than the standard graph-hypothetical methodology because of its capacity to deal with the qualities of edges viably. Theory of F-graphs is presently finding various applications in current science & innovation particularly in the fields of data hypothesis, neural framework, master frameworks, group examination, clinical finding, control hypothesis, etc. Numerous issues of down to earth intrigue can be displayed as coloring issues.

Coloring of the graph is a research zones of combinatorial enhancement because of its wide Apps, in actuality, viz. the executive's sciences, wiring printed circuits, asset allocation, booking issues, and so forth. A graph coloring issue is one of the most examined issues in discrete and combinatorial improvement. Proof of this can be found in different papers and books on graph coloring alongside related issues and guesses. F-graphs fuzzy coloring was characterized in 2004 by creators Onagh&Eslahchiand later created by them as coloring F-vertex in 2005. The coloring of vertex issue of a graph is the issue of allotting a color to every vertex so that the colors of its nearby vertices are extraordinary & colors quantity utilized is least.

Two kinds of coloring to be specific coloring of edges&coloring of vertex are normally connected with any graph. Coloring of edgesis a capacity that doles out to the edges with the goal that episode edges get various colors. We realize that graphs are a basic model of connection. A graph is an advantageous method of speaking to information including the connection within objects. Objects are spoken to by vertices and relations by edges. The essential graph coloring issue is to aggregate things in as not many gatherings as could reasonably be expected, subject to the imperative that no contradictory things end up in a similar gathering.

## II. DEFINITIONS

**Definition 1:** Given a F-graph $\tilde{G} = (V, \mu)$, itschromatic number of F-graph $x(\tilde{G}) = \{x, v(x))/x \in X\}$,where$X = \{1, \dots, |V|\}, v(x) = \sup\{\propto \in I/x \in A_{\propto}\}\ \forall x \in X\ and\ A_{\propto} = \{1, \dots., X_{\propto}\}\forall_{\propto}\in I$.

Issueof F-coloring comprises of deciding the F-graph chromatic number & a related coloring capacity. F-graph chromatic number is a standardized F-number whose modular worth is related with the vacant edge-set graph.

**Definition 2:** Let A be intuitionistic F-set (IFS) of a universe set X. Then $(\alpha, \beta) - cut\ of\ A$ is a crisp subset $C_{\alpha,\beta}(A)$of the IFS A is given by $C_{\alpha,\beta}(A) = \{x_i \in X | \mu_A(x_i) \geq \alpha, v_A(x_i) \leq \beta \in [0,1]\ with\ \alpha + \beta \leq 1$.

**Definition 3:** AF-set *defined* on a non-empty set $X$ is the Group$A = \{(X,\ \mu_A(x)/x \in X\}, where\ \mu_A: X \to I$is the enrollment work. In old-style theory of F-set, that is normally characterized by span [0, 1] with the end goal that

$$\mu_A(x) = \begin{cases} 0: x \notin X \\ 1: x \in X \end{cases}$$

It takes any moderate an incentive somewhere in the range of 0 and 1 speaks to the degree where $x\ \in\ A$. The set I could be discrete arrangement of the structure$I = \{0,1, \dots k\}\ where\ \mu_A(x) < \mu(x`)$shows that the level of enrollment of two is lower than the level of participation of x to $A$ is lower than the level of participation of x'.

**Definition4:** Let $G\ =\ (V, E)$be a graph. A vertex-coloring of G is a task of a color to every one of the vertices of G so that neighboring vertices are allocated various colors. On the off chance that the colors are looked over a lot of$k$ colors, at that point the vertex-coloring is known as an$k$ -vertex-coloring, condensed to$k$ -coloring, whether or not all $k$colors are utilized.

**Definition5:**Minor $k$, such that $G$ is $k$ −colorable, is known as the chromatic number of $G$, indicated by$X(G)$.

**Definition 6:** A family $\Gamma=\{\gamma_1, \gamma_2 \ldots .\gamma_k\}$ of F-sets on X is known as coloring of k-fuzzy of $G` = (X, \sigma, \mu)$ if

- $\gamma_i \wedge \gamma_j = 0,$
- for each solid edge $xy$ of G, $min\ \{\Upsilon_i(x), \Upsilon_i(y)\} = 0 (1 \le i \le k).$
- $\vee \Gamma = \sigma$

Minimal estimation of $k$ for which $G`$ has coloringof k−fuzzy, meant by $X^f(G`)$ is known as the chromatic F-number of $G`$. All through this, we use colors orange (O), blue (B), red (R), yellow (Y) & green (G) for coloring the graphs.

**Definition 7:** If G has a $k$-coloring, then $G$ is said to be $k$- colorable.

**Definition 8:** Minimal estimation of k for which $G$ has a fuzzy coloring, indicated by $X^f\ (G)$, is called the chromatic fuzzy number of $G$.

**Definition 9:** The $\alpha$F-graph cut characterized as $G_\alpha = (V_\alpha, E_\alpha, \sigma, \mu)$ in which $V_\alpha = \{v \in V | \mu \ge \alpha\}\ and\ E_\alpha = \{e \in E | \mu \ge \alpha\}$.

**Definition 10:** F-graph $\tilde{G} = (V, \mu)$, its F-number's chromatic number:

$$X(\tilde{G}) = \{(x, v(x))/x \in X\},$$

In which, $X = \{1, \ldots, |V|\}, v(x) = \sup\left\{\propto \in \frac{I}{x} \in A_\alpha\right\} \forall_x \in X\ and\ A_\propto = \{1, \ldots, \chi_\propto\} \forall \propto \in I.$

**Definition 11:** Leave G alone a straightforward graph. The chromatic polynomial of G is the quantity of ways we can accomplish an appropriate vertices coloring of G with the given $k$colors and it is meant by $P(G, k)$. It is a monic polynomial in k with whole number co-efcients, whose degree is the quantity of G's vertices.

**Definition 12:** The F-chromatic total of G meant by $\sum(G)$ is characterized as follows:

$$\sum(G) = min\ \{\textstyle\sum_\Gamma(G)/\Gamma\ is\ fuzzy\ colouring\}$$

The quantity of F-coloring of G is limited thus there exist a fuzzy $\Gamma_0$ which is called least F-coloring of G with the end goal that $\sum(G) = \sum_{\Gamma 0}(G)$.

## III. COLORING THEOREMS: FUZZY GRAPH THEORY

**Theorem 1:** Leave G a straightforward graph, and let G − e and G/e be the graphs gotten from G by erasing and getting an edge e. At that point,

$$P\ (G, k)\ =\ P\ (G\ -\ e, k)\ -\ P\ (G/e, k)$$

**Theorem 2:** Leave G be alone anF-graph and$\Gamma_0 = \{Y_1, Y_2, \ldots\ldots, Y_k\}$is least fuzzy entirety coloring of G. At that point,

$$\Sigma_{x\in C1}\theta_1(x) \geq \Sigma_{x\in C2}\theta_1(x) \geq \cdots \geq \sum_{x\epsilon CK}\theta_k(x)$$

**Theorem 3:** Leave G alone a fresh graph & H be G's sub-graph. At that point$\chi(H) \leq \chi(G)$.

**Theorem 4:** For an intuitionistic fuzzy graph $G^* = (X, E), \chi(G^*) = \chi^f(G^*)$.

**Proof:** Let $G^* = (X, E)$be an intuitionistic F-graph on n vertices, $\{x_1, x_2, \ldots, x_n\}$.

Let $\chi^f(G^*) = k$.

$\Leftrightarrow \Gamma = \{\gamma_1, \gamma_2, \ldots, \gamma_k\}$Is a k-fuzzy coloring and let$C_j$be the color relegated to vertices in$\Upsilon_j^*; j = 1, 2, \ldots, k$.

$\Leftrightarrow \{\gamma_1, \gamma_2, \ldots, \gamma_k\}$It is an intuitionistic F-group set in that:

$\gamma_j(xi) = \{(x_j, ?_1(x_j), \nu_1(x_j))\}\, S\, \{(x_i, ?_1(x_i), \nu_1(x_i)) \mid ?_2(x_i\, x_j) = 0, i\, 6 = j\}$.

$\Leftrightarrow \gamma_j^*$ It is independent arrangement of vertices for each$j = 1, 2, \ldots, k$.

$\Leftrightarrow \chi(G_{\alpha,\beta}^*) = k, G_{\alpha,\beta}^*$is the hidden fresh graph$G^*$. Now $\chi(G_{\alpha,\beta}^*) - \chi(G_t^*) = k\ where\ t = \min\{(\alpha, \beta) | \alpha, \beta \in [0, 1]\}$

$= max\{\chi_{\alpha,\beta} \mid \alpha, \beta \in [0, 1]\}$Also, henceforth the confirmation.

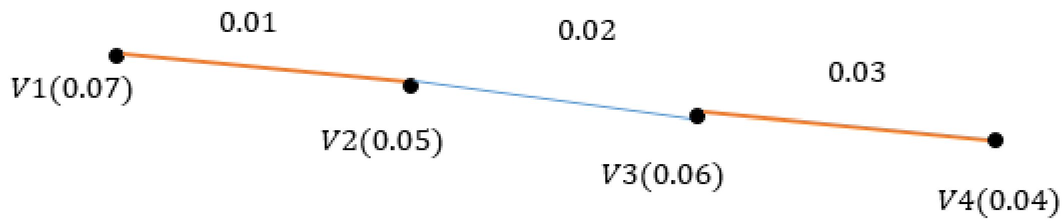
**Theorem 5:** Let $G = (V, \mu)$be anF-graph with F-edges & fresh vertices. At that point the level of $P_\propto^f(G, k) = |V|, for\ all \propto \in I$.

**Proof:** Let $G = (V, \mu)$be aF-graph with F-edges&fresh vertices. Presently,$G_\propto = (V, E_\propto), where\ E_\propto = \{(u, v)|\mu(u, v) \geq\propto\}$. we realize that the level of the chromatic polynomial of a fresh graph$G_\propto$is the vertices quantity of$G_\propto$that is, the level of$P(G_\propto, k) = |V|$. $P_\propto^f(G, k) = P(G_\propto, k), for \propto \in$ I. from this it follows that the level of $P_\propto^f(G, k)$is equivalent to the level of$P(G_\propto, k)$. Along these lines, $P_\propto^f(G, k) = |V|$.

**Theorem 6:** On the off chance that G is an F-enchantment way of n vertices, $\grave{\chi}_{fm}(G) = 2$.

**Proof:** Leave G alone anF-enchantment graph with the end goal that$G^*$ is a fuzzy enchantment way of n vertices. Plainly every edge of G isn't solid. Since G* is a fuzzy enchantment graph. So we can give color R &W on the other hand to the edges of G. This is a legitimate fuzzy edge coloring.

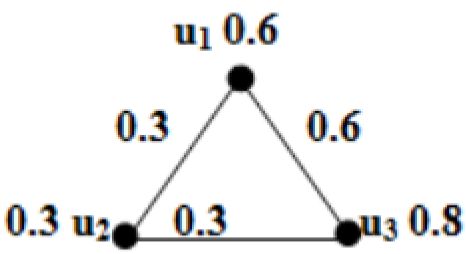
Hence, $\grave{\chi}_{fm}(G) = 2$.




**Theorem 7:** The fuzzy dominator coloring of a Cartesian result of complete fuzzy graph G×H is m+n-1 where n &mare the vertices quantity of complete fuzzy graph G and H.

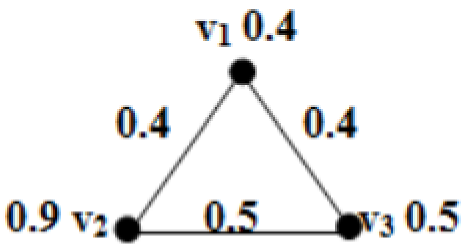
**Proof:** As each pair of vertices is unequivocally adjoining and the solid level of every vertex is$m + n - 1$ in $G \times H. In\ G \times H$, there exists an m complete fuzzy sub-graph with n vertices. Every vertex in complete fuzzy sub-graph commands itself and they overwhelm one vertex in m sub-graphs. So the m prevailing vertices allot m colors and the solid neighbor of predominant vertices in each total fuzzy sub-graph appoint n-1 color. In this way, $\chi_{fd}(G \times H) = m + n - 1$.

**G:**



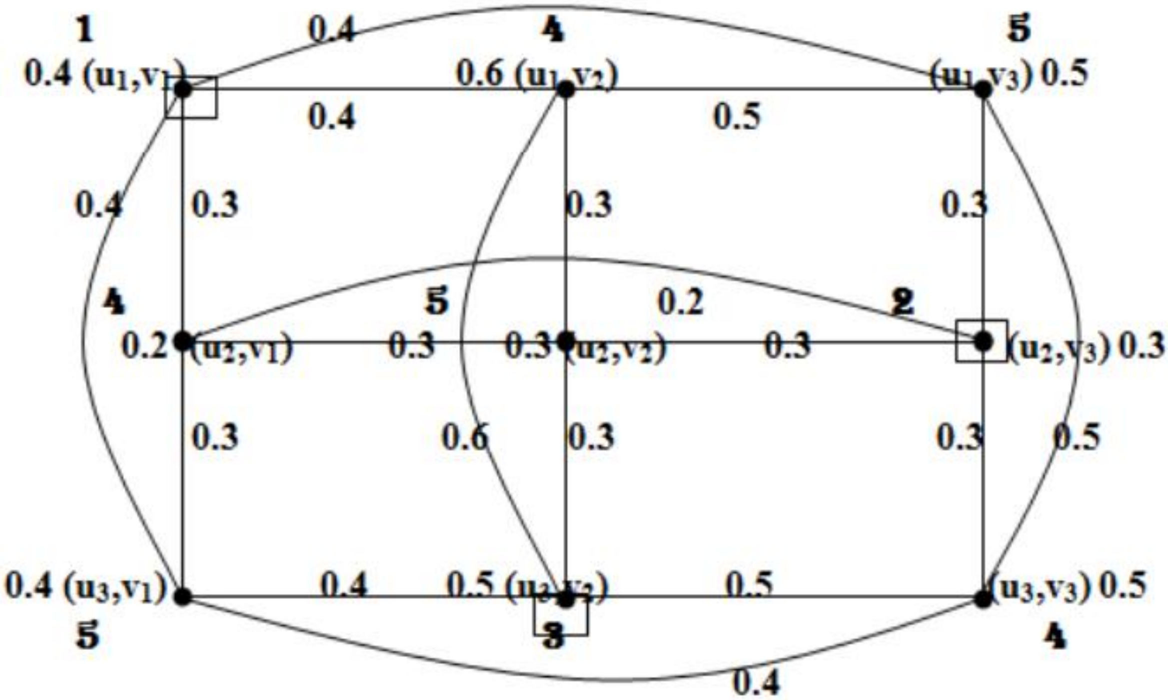

***$K_3$ Fuzzy Graph***

**H:**




***$K_3$ Fuzzy Graph***

$G \times H$




$K_3 \times K_3$ ***Complete Fuzzy Graph Cartesian Product***

## IV. CONCLUSION

Graph theory is an amazingly valuable device for understanding various territories. Since the exploration of demonstrating of certifiable issues frequently includes multi-specialist, multi-object, multi-list, multi-polar information. F-graphs coloring assumes a fundamental job in the pragmatic &theory applications. A graph in this setting is comprised of hubs, vertices, or focuses that are associated with edges, bends, or lines. The coloring issue comprises of deciding the chromatic number of a graph & a related coloring capacity. Graph might be undirected, implying that there is no differentiation among2 vertices related with each edge, or its edges might be guided starting with 1 vertex then onto the next. The new idea of coloring capacity considers a contrariness degree identified with the estimation of the enrollment work on each edge.

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