

RETRENCHMENT MANPOWER MODELS IN A SINGLE GRADE SYSTEM

J.Princyvishvamalar

Dhanalakshmi Srinivasan College of Arts and Science for Women(A), Perambalur

M.Manjumari

Dhanalakshmi Srinivasan College of Arts and Science for Women(A), Perambalur

Abstract

In any organisation retrenchment is necessary whenever number of employee are more than required and number of jobs are less than required. Due to this unnecessary expenditure, the growth of the organisation will get affected and may lead to complete breakdown of the organisation. In order to avoid this situation, retrenchment have to be done by checking the job strength and staff strength randomly or periodically. An attempt has been made to study the problem of time to retrenchment for an organisation and the explicit expressions for expected time to retrenchment and variance of time to retrenchment have been derived for a single graded organisation.

Keywords- Single grade, staff strength, job strength, retrenchment, checking points, threshold.

AMS Mathematics Subject Classification (2010)- 990B70, 91B40, 91D35, 60H30.

I. INTRODUCTION

In [3], the reason for retrenchment and effects of retrenchment are discussed. In [2], termination by retrenchment has been studied. Retrenchment models for single graded organisation is studied using two retrenchment policies namely univariate policy of retrenchment and bivariate policy of retrenchment. Assuming the staff strength and job strength as independent and identically distributed geometric random variables and inter-checking

points as independent and identically distributed exponential random variables, the mean and variance of time to retrenchment are derived.

II. Model 1: Retrenchment model for staff strength

A. Assumptions

- Staff strength at any checking point is linear.
- Staff strength and inter-checking points (time between $i-1^{\text{th}}$ and i^{th} checking point) are statistically independent.

UNIVARIATE POLICY OF RETRENCHMENT FOR STAFF STRENGTH:

Retrenchment is done at the checking point whenever the staff strength is greater than the threshold (the staff strength required for the organisation to function)

B. Notations

i : checking point for staff strength in an organisation in $[0, \infty)$, $i=1,2,\dots$

U_i : independent and identically distributed (i.i.d)

exponential random variable representing inter-checking point, $i=1,2,\dots$ with parameter $(\alpha > 0)$.

R_i : waiting time upto i^{th} checking point.

W : time for retrenchment in an organisation.

X_i : independent and identically distributed geometric

random variable representing staff strength in an

organisation $i=1,2,\dots$ in the i^{th} checking point with parameter $(\theta > 0)$.

c_1 : threshold for staff strength, c_1 is a positive constant

$\chi(A)$: indicator function of the event A.

$X_i \leq c_1 < X_{i+1}$: Event that the threshold for staff strength lies between the staff strength at i^{th}

checking point and $i + 1^{th}$ checking point.

$E(W)$: Mean time to retrenchment.

$E(W^2)$: Second moment of time to retrenchment

$V(W)$: Variance of time to retrenchment

$$V(W) = E[W^2] - [E(W)]^2$$

C. .Main Result

For $i = 1, X_1$ is the staff strength in the 1st check. U_1 is the time at which first check is made. If X_1 is greater than c_1 then go for retrenchment. If not then retrenchment is not done at first check. And retrenchment will be done when X_i is greater than c_1 in the any of the subsequent i^{th} check for staff strength.

$$W = \sum_{i=1}^{\infty} R_{i+1} \chi(X_i \leq c_1 < X_{i+1}) \quad (1)$$

Taking expectation on both sides of (1) and using the result in [1], we get

$$E(W) = \sum_{i=1}^{\infty} E(R_{i+1}) P(X_i \leq c_1 < X_{i+1}) \quad (2)$$

Since $R_{i+1} = \sum_{j=0}^i U_{j+1}$, from (2), we get

$$E(W) = \sum_{i=1}^{\infty} \sum_{j=0}^i E[U_{j+1}] P(X_i \leq c_1 < X_{i+1}) \quad (3)$$

Since U_j 's are i.i.d exponential random variables

$$E[U_{j+1}] = E[U], \quad j = 0, 1, 2, \dots \quad (4)$$

Using (4) in (3)

$$E(W) = \sum_{i=1}^{\infty} \sum_{j=0}^i E[U] P(X_i \leq c_1 < X_{i+1}) \quad (5)$$

$$E(W) = \sum_{i=1}^{\infty} E[U] (i+1) P(X_i \leq c_1 < X_{i+1}) \quad (6)$$

As $P(X_i \leq c_1 < X_{i+1}) = P(X_i \leq c_1) P(X_{i+1} > c_1)$

and on simplification,

$$E(W) = \frac{1}{\alpha} \sum_{i=1}^{\infty} (i+1) (\bar{\theta}^{c_1} - \bar{\theta}^{2c_1}) \quad (7)$$

(7) gives mean time to retrenchment for this model.

$$W^2 = [\sum_{i=1}^{\infty} R_{i+1} \chi(X_i \leq c_1 < X_{i+1})]^2 \quad (8)$$

Since $\chi^2(A) = \chi(A)$, (8) becomes

$$W^2 = \sum_{i=1}^{\infty} R_{i+1}^2 \chi(X_i \leq c_1 < X_{i+1}) \quad (9)$$

Now

$$\begin{aligned} E(R_{i+1}^2) &= V(R_{i+1}) + E^2(R_{i+1}) \\ &= V(\sum_{j=0}^i U_{j+1}) + E^2(\sum_{j=0}^i U_{j+1}) \end{aligned}$$

$$\text{Hence } E(R_{i+1}^2) = (i+1)V(U) + (i+1)^2[E(U)]^2 \quad (10)$$

By taking expectation on both sides of (9) and using (10) we get

$$\begin{aligned} E(W^2) &= V(U) \sum_{i=1}^{\infty} (i+1) P(X_i \leq c_1) P(X_i > c_1) \\ &\quad + E(U)^2 \sum_{i=1}^{\infty} (i+1)^2 P(X_i \leq c_1) P(X_i > c_1) \end{aligned}$$

$$\begin{aligned} E(W^2) &= \frac{1}{\alpha^2} \sum_{i=1}^{\infty} (i+1) \left(\bar{\theta}^{-c_1} - \bar{\theta}^{-2c_1} \right) \\ &\quad + \frac{1}{\alpha^2} \sum_{i=1}^{\infty} (i+1)^2 \left(\bar{\theta}^{-c_1} - \bar{\theta}^{-2c_1} \right) \quad (11) \end{aligned}$$

Using (7) and (11) we get the explicit expression for variance of time to retrenchment.

III. Model 2: Retrenchment model for job strength

A. Assumptions

- Job strength at any checking point is linear
- Job strength and inter-checking points are statistically independent.

UNIVARIATE POLICY OF RETRENCHMENT FOR JOB STRENGTH

Retrenchment is done at the checking point whenever the job strength is lesser than the threshold (the job strength required for the organisation to function).

B. Notations

Notations are same as in section 1.1 except

J_i : independent and identically distributed geometric random variable representing job strength in an organisation in the i^{th} checking point $i=1, 2, \dots$ with parameter ($\gamma > 0$).

c_2 : threshold for job strength.

c_2 is a positive constant.

$(J_i \geq c_2 > J_{i+1})$: Event that the threshold for job

strength lies between the job strength at i^{th}

checking point and $i + 1^{th}$ checking point.

C. Main Result

For $i = 1$, let X_1 be the job strength at the 1st check. U_1 be the time at which first check is made. If J_1 is lesser than c_2 then go for retrenchment. If not then retrenchment is not done at first check. And retrenchment

will be done when J_i is lesser than c_2 in the any of the subsequent i^{th} check for job strength.

$$W = \sum_{i=1}^{\infty} R_{i+1} \chi(J_i \geq c_2 > J_{i+1})$$

Taking expectation on both sides and using the result in [1], we get

$$E(W) = \sum_{i=1}^{\infty} E(R_{i+1}) P(J_i \geq c_2 > J_{i+1})$$

Proceeding as in section 2.3 and on simplification

$$E(W) = \left[\frac{1}{\alpha} \sum_{i=1}^{\infty} (i+1) (\bar{\gamma}^{c_2} - \bar{\gamma}^{2c_2}) \right] \quad (12)$$

(12) gives mean time to retrenchment for this model.

$$W^2 = \left[\sum_{i=1}^{\infty} R_{i+1}^2 \chi(J_i \geq c_2 > J_{i+1}) \right] \quad (13)$$

Proceeding similar to section 2.3,

$$E[W^2] = \left[\left(\frac{1}{\alpha} \right)^2 \sum_{i=1}^{\infty} (i+1) (\bar{\gamma}^{c_2} - \bar{\gamma}^{2c_2}) \right] + \left[\left(\frac{1}{\alpha^2} \right) \sum_{i=1}^{\infty} (i+1)^2 (\bar{\gamma}^{c_2} - \bar{\gamma}^{2c_2}) \right] \quad (14)$$

Using (12) and (14) variance of time to retrenchment for this model can be obtained.

IV. Model 3: Retrenchment model for staff and job strength

A. Assumptions

- Staff strength and job strength at any checking point is linear.
- Staff strength, job strength and inter-checking

points are statistically independent.

BIVARIATE POLICY OF RETRENCHMENT

FOR STAFF STRENGTH AND JOB STRENGTH:

Retrenchment is done at the checking point when the staff strength greater than the threshold for staff strength as well as job strength lesser than the threshold for job strength.

B. Notations

Same as in section 2.2 and section 3.2

C. Main result

If X_1 is greater than c_1 and J_1 is lesser than c_2 then go for retrenchment. If not then retrenchment is not done at the first check. And retrenchment will be done when X_i is greater than c_1 and J_i is lesser than c_2 in the any of the subsequent i^{th} check for both staff and job strength.

$$W = \sum_{i=1}^{\infty} R_{i+1} \chi(X_i \leq c_1 < X_{i+1}, J_i \geq c_2 > J_{i+1}) \quad (15)$$

Taking expectation on both sides of (15) and since X_i and J_i are independent

$$E(W) = \sum_{i=1}^{\infty} E(R_{i+1}) P(X_i \leq c_1) P(X_{i+1} > c_1) P(J_i \geq c_2) P(J_{i+1} < c_2) \quad (16)$$

$$E(W) = \frac{1}{\alpha} \sum_{i=1}^{\infty} (i+1) \left(\theta^{-c_1} - \theta^{-2c_1} \right) \left(\gamma^{-c_2} - \gamma^{-2c_2} \right) \quad (17)$$

(17) gives mean time to retrenchment for this model

$$W^2 = \sum_{i=1}^{\infty} R_{i+1}^2 \chi(X_i \leq c_1 < X_{i+1}, J_i \geq c_2 > J_{i+1})$$

By taking expectation on both sides and on simplification we get,

$$E(W^2) = \left[V(U) \sum_{i=1}^{\infty} (i+1) + E(U)^2 \sum_{i=1}^{\infty} (i+1)^2 \right] \times P(X_i \leq c_1) P(X_i > c_1) P(J_i \geq c_2) P(J_{i+1} < c_2) \quad (18)$$

$$E(W^2) = \left[\frac{1}{\alpha^2} \sum_{i=1}^{\infty} (i+1) + \frac{1}{\alpha^2} \sum_{i=1}^{\infty} (i+1)^2 \right] \left(\theta^{-c_1} - \theta^{-2c_1} \right) \left(\gamma^{-c_2} - \gamma^{-2c_2} \right)$$

Using (17) and (18) we obtain the expression for variance of time to retrenchment for this model.

V. Numerical illustration and conclusion

The influence of parameters on the performance measure namely mean time to retrenchment is studied numerically.

In the following table the performance measure is calculated by varying the parameters α and θ and keeping threshold c_1 fixed.

Table 1: Effect of α and θ on performance measure E(W) in Model 1 ($c_1 = 100$)

α	θ	E(W)
0.1	0.0090	48.192
0.1	0.0083	49.143
0.1	0.0076	49.773
0.2	0.0090	24.096
0.2	0.0083	24.572
0.2	0.0076	24.887
0.3	0.0090	16.064
0.3	0.0083	16.381
0.3	0.0076	16.591

From the above table we observe that as θ decreases, the mean time to retrenchment increases.

In the following table the performance measure is calculated by varying the parameters α and γ and keeping threshold c_2 fixed.

Table 2: Effect of α and γ on performance measure E(W) in Model 2 ($c_2 = 50$)

α	γ	E(W)
0.1	0.022	44.139
0.1	0.028	36.658
0.1	0.04	22.603
0.2	0.022	22.069
0.2	0.028	18.329
0.2	0.04	11.302
0.3	0.022	14.713
0.3	0.028	12.219
0.3	0.04	7.5344

From table 2, we observe that as γ increases the mean time to retrenchment decrease.

In the following table the performance measure is calculated by varying the parameters α , θ and γ and keeping thresholds c_1 and c_2 fixed.

Table 3: Effect of α , θ and γ on performance measure E(W) in Model 3
($c_1 = 100$ and $c_2 = 50$)

α	θ	γ	E(W)
0.1	0.0090	0.022	10.636
0.1	0.0083	0.028	9.0075
0.1	0.0076	0.04	5.6251
0.2	0.0090	0.022	5.3178
0.2	0.0083	0.028	4.5038
0.2	0.0076	0.04	2.8126
0.3	0.0090	0.022	3.5452
0.3	0.0083	0.028	3.0025
0.3	0.0076	0.04	1.8750

From table 3, we observe that as θ decreases and γ increases, the mean time to retrenchment decreases.

In all the three models, mean time to retrenchment decreases as α increases.

VI. REFERENCES

- [1] Cormen Thomsan H, Leiserson Charles E, Rivest Ronald L, Stein Clifford (2009),

“Introduction to Algorithms” MIT press, London, England, pp-118-119.

- [2] Josephat Stephan Itika (2011) “Fundamentals of human resource management” African studies centre, Leiden, Muzumbe university, p.228

- [3] Roesmary Boateng Coffie (2016) “Retrenchment process and the coping mechanisms of Employing from the Banking Industry in Ghana: A case of Adot Bank Ghana Limited”, Ph.D., Thesis, Kwame Nkrumah University of Science and Technology, pp.16-29.